Math 676. Algebraic Number Theory

Instructor: Prof. Brian Conrad, `bdconrad@math.lsa.umich.edu`
Office: 2856 East Hall
Office hours: MWF, 10–11am, and by appointment.

Prerequisites: Math 593 and 594 (PID's, tensor products, and Galois theory – including positive characteristic), and Math 575.

Textbooks: There is no required text, but some books related to the course material will be kept on reserve at the library: Lang's *Algebraic Number Theory*, Cassels-Frölich's *Algebraic Number Theory*, Serre's *Local Fields*, Fröhlich-Taylor's *Algebraic Number Theory*, Samuel's *Algebraic Theory of Numbers*, Marcus' *Number Fields*, and Koblitz' *p-adic numbers, p-adic analysis, and zeta functions.*

Homework/exams: There will be no exams, but there will be weekly homework (for the student to make sure they understand what is going on) and students are strongly encouraged to fill in details omitted in lecture. The grade will be based entirely on the weekly homeworks, and the lowest homework grade will be dropped (and all others count equally). Late homeworks will not be accepted for any reason.

Course description: The fundamental theorem of arithmetic states that any non-zero integer is expressible as a product of primes in a manner that is unique up to order of the factors and units (i.e., signs). Algebraic number theory begins with trying to understand how this generalizes (or fails to do so) in algebraic number fields. An *algebraic number field* is a finite extension of $\mathbf{Q}$, and an element of such a field is called an *algebraic number*. This course covers the basic structure of such fields and some analogues in positive characteristic. In particular, we will study algebraic integers (these play the role analogous to that of $\mathbf{Z}$ inside of $\mathbf{Q}$) and we will prove two of the important *finiteness theorems*: finiteness of class groups (these measure the failure of the fundamental theorem of arithmetic for rings of algebraic integers) and finite-generatedness of unit groups (this generalizes classical results centered on Pell's equation).

We will include as many examples and applications as possible, and some of these will be developed in the homework. The main topics to be covered are:

(1) Dedekind domains, global fields, rings of integers.
(2) Valuations and local fields.
(3) Class groups
(4) Unit groups
(5) L-functions and class number formulas.

If time permits we may discuss the adele ring and idele group attached to a global field.

This course (or at least a solid command of the material in it, ignoring characteristic p if you are analytically-inclined) is absolutely essential for anyone wishing to study number theory. It should also be useful and interesting for those studying algebraic geometry.