# Math 249B. Class Field Theory

**Instructor**: Prof. Brian Conrad, `conrad@math.stanford.edu`
**Office**: 383CC, Sloan Hall.
**Office hours**: MWF, 10–11am.

**Prerequisites**: Math 248A or knowledge of basic algebraic number theory (local and global fields, and adeles).

**Textbooks**: There is no required text, but some books related to the course material will be kept on reserve at the library. Recommended books include Lang's *Algebraic Number Theory*, Neukirch's *Algberaic Number Theory*, Cassels-Frölich's *Algebraic Number Theory*, Serre's *Local Fields*, Artin-Tate's *Class Field Theory*, and Serre's *Algebraic Groups and Class Fields*. These last two are actually pretty tough, and probably shouldn't be looked at until after the course is over (if ever).

**Homework/exams**: There will be no exams or collected homework, but I will post homework every week which you should absolutely make a serious attempt to complete (or at least think about for a while) since the subject is a difficult one to learn and hence active engagement with the concepts in both theoretical and concrete settings is imperative for success. There will also be some handouts to supplement the lectures. Feel free to ask me in office hours or elsewhere about any questions you have concerning material in the homework (or handouts or lectures).

**Course description**: Class field theory is one of the great triumphs of number theory from the first half of the 20th century, and gives a complete description of the abelian extensions of global fields and local fields. There are many ways to approach the theory (using adeles, group cohomology, algebraic geometry, etc.), and knowledge of these different points of view is important in order to fully understand the theory. However, the proofs of class field theory are quite indirect and this state of affairs was perhaps best described by Tate: the theory is true because it could not be otherwise.

While it is somewhat instructive to know what goes into the proofs of the main theorems (e.g., to see what obstacles prevent the proofs from being entirely constructive), it cannot be said that the grungy details of these proofs are particularly relevant to using the theory in practice. Thus, in the first half of the course we will emphasize an understanding of the statements of the main results (in their many different forms) and will not place much emphasis on how the main theorems are proven; precise references will be given for those who wish to read the details of the proofs of the main theorems. Once we have spent some time digesting what class field theory tells us, we will study some applications of the theory, such as in the context of imaginary quadratic fields and abelian coverings of algebraic curves.

An understanding of class field theory in its modern adelic form is an essential first step in the direction of the Langlands Program, to be discussed in Math 249C.