MATH 116, SPRING 2004
HOMEWORK 4 SOLUTIONS

Pg. 44, 1:

$$\sin i = \frac{e - e^{-1}}{2} i \tag{1}$$

$$\cos i = \frac{e + e^{-1}}{2} \tag{2}$$

$$\begin{aligned} \tan(1+i) &= \frac{\sin(1+i)}{\cos(1+i)} \qquad (3) \\ &= \frac{\cos(1)\sin(i) + \sin(1)\cos(i)}{\cos(1)\cos(i) - \sin(1)\sin(i)}. \qquad (4) \end{aligned}$$

The values in the final expression have been calculated.

Pg. 44, 3:

$$\cos(x+iy) = \cos(x)\cos(iy) - \sin(x)\sin(iy) \tag{5}$$

$$= cos(x)\frac{e^{-y}+e^{y}}{2} + \sin(x)\frac{e^{-y}-e^{y}}{2}i. \tag{6}$$

Pg. 47, 3:

Using the formula

$$e^{iz} = \cos z + i \sin z, \tag{7}$$

we obtain

$$e^{-\frac{\pi i}{2}} = \cos(-\frac{\pi}{2}) + i \sin(-\frac{\pi}{2}) \tag{8}$$
$$= -i. \tag{9}$$

The other values are handled in the same manner.

Pg. 47, 4:

We will solve for the value $1+i$. The others are similar.

$$e^{iy} = \cos y + i \sin y, \tag{10}$$

so

$$e^{x+iy} = 1+i \tag{11}$$

precisely when

$$e^x \cos y = 1 \tag{12}$$

and

$$e^x \sin y = 1. \tag{13}$$

We can equate these to find that we require

$$e^x \sin y = e^x \cos y. \tag{14}$$

$y = \frac{\pi}{4} + 2\pi k$ for any integer k works (keep in mind that $y = \frac{\pi}{4} + \pi k$, k odd doesn't work because of $e^x \cos y = 1$). We set $x = \log(\frac{2\sqrt{2}}{2})$.