MATH 116, SPRING 2004
PRACTICE EXAM
SUGGESTIONS AND ANSWERS FOR SELECTED PROBLEMS

It is suggested that you work on these problems by yourself and only refer to these hints and answers when required.

1.

Answers:

(a) $-4 + 2\pi i$

(b) $4(e^{\pi} - 1)$

3.

Suggestion:

If you estimate the absolute value of the integrand and just plug in the expression for the curve, you'll obtain the estimate via the triangle inequality.

4.

Suggestion:

These are just straightforward Cauchy formula problems. For (b), be careful about where the zeros of the denominator land with respect to the curve. For (c), make sure you put the denominator in the standard form.

Answers:

(a) 2π

(b) $\pi i/4$

(c) $-\pi i/2$

5, 6.

These are in the book in the section where Liouville's theorem is proved. Number 5 is just a special case of Liouville's theorem.

7.

Suggestion:

You can find the order of poles by checking the limit of $(z-a)^k f(z)$ as z goes to a for various k (the order will be the $k-1$ for the smallest k that works), or you can just read off the order for rational functions.

For these problems, you can find the residues by using Cauchy's formula to evaluate the integral which occurs in the Residue Theorem.

8.

Suggestion:

The first problem is just like number 7. For the others, expanding into Laurent series provides quick solutions.

9.

Suggestion:

Again, as in number 7, you can just use Cauchy's formula to find the residues. Don't be surprised if your calculations get a little messy!

10.

Suggestion:

Using power series methods or Cauchy's formula as usual, the residues can be determined. Then the Residue Theorem tells us to just add them up and multiply by 2π. All of the winding numbers are 1 here.

Answers:

(a) $-2\pi i$

(b) $2\pi i$

(c) $10\pi i$

11.

Suggestion:

For the first two, you can just use a semi-circle and the Residue Theorem. The integrals along the curved part of the semi-circle will go to zero as R goes to infinity. For the third integral, notice that the integrand is just the imaginary part of $\frac{xe^{2ix}}{x^4+2}$. Consult your text for helpful suggestions (there is a section on integrals of the form $\int R(x)e^{ix}$).

Answers:

(a) $\pi/4$

(b) $\pi/6$

12.

Suggestion:

You can use Rouche's Theorem. Pick the part of the polynomial with the largest coefficient and use it to dominate the rest.