TOPOLOGY SEMINAR

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Infinite loop spaces and positive scalar curvature

Tuesday, April 9th, 4:00pm, in 383-N

Abstract: It is well known that there are topological obstructions to a manifold M admitting a Riemannian metric of everywhere positive scalar curvature (psc): if M is Spin and admits a psc metric, the Lichnerowicz–Weitzenböck formula implies that the Dirac operator of M is invertible, so the vanishing of the $\hat{A}$ genus is a necessary topological condition for such a manifold to admit a psc metric. If M is simply-connected as well as Spin, then deep work of Gromov–Lawson, Schoen–Yau, and Stolz implies that the vanishing of (a small refinement of) the $\hat{A}$ genus is a sufficient condition for admitting a psc metric. For non-simply-connected manifolds, sufficient conditions for a manifold to admit a psc metric are not yet understood, and are a topic of much current research.

I will discuss a related but somewhat different problem: if M does admit a psc metric, what is the topology of the space $\mathcal{R}^+(M)$ of all psc metrics on it? Recent work of V. Chernysh and M. Walsh shows that this problem is unchanged when modifying M by certain surgeries, and I will explain how this can be used along with work of Galatius and the speaker to show that the algebraic topology of $\mathcal{R}^+(M)$ for M of dimension at least 6 is as complicated as can possibly be detected by index-theory.