TOPOLOGY SEMINAR

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Tits buildings, class groups, and the high-dimensional cohomology of $\text{SL}_n\mathcal{O}_K$

Tuesday, September 30th, 4:00pm, in 383-N

Abstract: Let $K$ be a number field with class number 1. We prove that if $K$ admits a real embedding $K \hookrightarrow \mathbb{R}$, then $H^{vcd}(\text{SL}_n\mathcal{O}_K;\mathbb{Q}) = 0$, where $vcd$ is the virtual cohomological dimension of the lattice $\text{SL}_n\mathcal{O}_K$.

To do this, we prove that the homology of the Tits building for $\text{SL}_nK$ is generated by $\mathcal{O}_K$-integral apartments; in fact, this holds for a wide class of Dedekind domains of arithmetic type, including $\mathbb{Z}[i,\frac{1}{3}] \subset \mathbb{Q}(i)$ and $\mathbb{F}_q[T^\pm 1] \subset \mathbb{F}_q(T)$. The key technical ingredient is that the complex of partial bases for $\mathcal{O}_K^n$ (previously used in proofs of homological stability) is Cohen–Macaulay.

In contrast, we prove that the codimension-0 Betti number of $\text{SL}_n\mathcal{O}_K$ grows exponentially when the class number of $K$ is greater than 2. Joint work with Benson Farb and Andrew Putman.