TOPOLOGY SEMINAR

# Thomas Church (Stanford)

## Combinatorial stability and representation stability

Tuesday, April 2nd, 4:00pm, in 383-N

*Abstract*: If you choose a squarefree polynomial $f(T) \in \mathbb{F}_q[T]$ uniformly at random, it will have slightly less than one linear factor on average. The exact value of this expectation depends on the degree of $f(T)$, but as $\deg f(T) \to \infty$ the expectation stabilizes and converges to

$$1 - \frac{1}{q} + \frac{1}{q^2} - \frac{1}{q^3} + \cdots = \frac{1}{1 + \frac{1}{q}}.$$

In joint work with J. Ellenberg and B. Farb, we proved that the stabilization of this combinatorial formula, and other statistics like it, is equivalent to a representation-theoretic stability in the cohomology of braid groups. I will describe how combinatorial stability for statistics of squarefree polynomials, of tori in $\mathrm{GL}_n(\mathbb{F}_q)$, and other natural geometric counting problems can be converted to questions of representation stability in topology, and vice versa.

This talk is intended for a general mathematical audience, and should be accessible to graduate students in any field.