Math 196-47, Mr. Church

Some sample problems for the second midterm;
or, My thinking on what questions are reasonable to ask.

Appropriate:

- Use Cramer's rule to find the solution to $A\vec{x} = \vec{b}$ (with explicit matrices/vectors, not letters).
- Assume that S is a collection of vectors in $\mathbb{R}^n$. State the mathematical definition of the phrase "S is a subspace".
- Given some (explicit) set S, determine whether it is a subspace.
- Are these (explicit) vectors linearly independent? Do they span $\mathbb{R}^n$? Are they a basis for $\mathbb{R}^n$?
- Assume that S is a subspace of $\mathbb{R}^n$. State the definition of the dimension of S.
- Let A be an $m \times n$ matrix. State the definition of the row space (or column space, or null space) of A. Find a basis.
- Find a basis for the subspace $\{(2t - u, 3t, t + u, t, u)\}$.
- State the triangle inequality.
- Compute the angle between the vectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.
- You may need to use the "bilinearity" rules of inner product

  $$\text{e.g. } \langle \vec{v}, \vec{w_1} + \vec{w_2} \rangle = \langle \vec{v}, \vec{w_1} \rangle + \langle \vec{v}, \vec{w_2} \rangle.$$

- Let $\vec{v}$ be the vector in the plane

  $$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

  Compute the vector obtained from $\vec{v}$ by rotating it $135°$ counterclockwise.
- Are these vectors perpendicular? Compute the projection of this vector onto that vector.
- Find a basis for the subspace $\{v_1, v_2, v_3\}^\perp$ (given explicit vectors).
- Is this matrix orthogonal? Is this collection of vectors an ortho**normal** basis?
- Prove that an orthogonal matrix can never have 2 as an entry.

Inappropriate: (Mostly by "inappropriate" I mean "too proof-y")

- HW9, question 2.
- Prove that every subspace has a basis.
- Prove that a matrix preserving lengths preserves inner products.
- Anything that requires row-reducing a 4×4 matrix (I already know you can do that).