Math 196-47, Mr. Church, Homework 3

Due at the beginning of class on Friday, April 17.

Please staple your homework.

1. Find the solution sets of the following systems of linear equations.

   (a)
   $$\begin{array}{rrrl} x & & +z & = 0 \\ & 2y & -z & = 0 \\ x & -2y & +2z & = 0 \end{array}$$

   (b)
   $$\begin{array}{rrrl} x & & +z & = 0 \\ & 2y & -z & = 1 \\ x & -2y & +2z & = 1 \end{array}$$

   (c)
   $$\begin{array}{rrrl} x & & +z & = 0 \\ & 2y & -z & = 1 \\ x & -2y & +2z & = 0 \end{array}$$

   (d) Describe the relation between the solution set of (a) and the solution sets of (b) and (c).

2. Determine whether each system has a solution, and if so whether the solution is unique, by computing the ranks of the associated matrix and of the augmented matrix. (Theorem 3.4.7 is what we're using here.)

   (a)
   $$\begin{array}{rrrl} 2x & +y & -z & = 2 \\ & y & -2z & = -1 \\ & -2y & +4z & = 2 \\ x & +y & +z & = 3 \end{array}$$

   (b)
   $$\begin{array}{rrrl} a & -2b & +c & = -1 \\ -2a & +b & +c & = 1 \\ a & -5b & +4c & = -2 \end{array}$$

   (c)
   $$\begin{array}{rrl} 2x & +y & = 1 \\ -4x & -2y & = 3 \end{array}$$

3. "**A system of linear equations can never have exactly two solutions.**"
   In no more than a few sentences, use what we have learned about systems of equations to explain why this is true.