Math 196-47, Mr. Church, "Homework" 14
Not to be turned in or graded.

Here are some problems that might have appeared on Homework 14, had it existed. This is longer than a real assignment; of course, you don't have to turn it in, so that's not unfair.I don't expect you do all of these problems (or any, except if you wan to study the material from Section 8.3).

1. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}3\\2\end{pmatrix}$, $T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$.
   (a) Find the standard matrix $[T]$.
   (b) Let $\mathcal{B} = \left\{\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}\right\}$. Find $[T]_{\mathcal{B}}$.
2. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}4\\2\end{pmatrix}$, $T\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}-4\\2\end{pmatrix}$.
   (a) Find the standard matrix $[T]$.
   (b) Let $\mathcal{B} = \left\{\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}-1\\1\end{pmatrix}\right\}$. Find the associated coordinate-change matrix P.
   (c) Find $[T]_{\mathcal{B}}$.
   (d) Using the coordinate-change matrix P, compute $\begin{pmatrix}3\\3\end{pmatrix}_{\mathcal{B}}$, $\begin{pmatrix}4\\2\end{pmatrix}_{\mathcal{B}}$, and $\begin{pmatrix}1\\3\end{pmatrix}_{\mathcal{B}}$.
3. Given $T\colon \mathbb{R}^n \to \mathbb{R}^n$ and $\mathcal{B} = \{v_1, \ldots, v_n\}$ some basis for $\mathbb{R}^n$, show that (or at least understand why):

   $[T]_{\mathcal{B}}$ is diagonal if and only if each v_i is an eigenvector of T.

All the following questions are good exercise (and may be easier than Question 2 above). Exercise 8.3.1, 8.3.3, and 8.3.5 have answers in the back of the book.

4. Exercise 8.3.1.
5. Exercise 8.3.2.
6. Exercise 8.3.3 (intuitively this should make sense).
7. Exercise 8.3.4.
8. Exercise 8.3.5 (although I found part (b) a bit confusing notationally).
9. Exercise 8.3.6 (for part (c), this is just the coordinate-change matrix P).

The "Group project: Matrix representations from transformation properties" covers similar material, but goes a bit farther afield and is not essential. The last sentence of (a) is a bit more abstract, but might be good practice if you have trouble working with linear transformations.