Homework 5

Math 120 (Thomas Church, Spring 2018)

Due Thursday, May 3 at 11:59pm.

Recall that a group G is *simple* if it has no normal subgroups except itself and $\{1\}$.

Question 1. Prove that if $|G| = 312 = 2^3 \cdot 3 \cdot 13$ then G is not simple.

Question 2. Suppose that G is a simple group with $|G| = 168 = 2^3 \cdot 3 \cdot 7$.
How many elements of order 7 does G contain? Justify your answer.

Question 3. Prove that if $|G| = 56 = 2^3 \cdot 7$ then G is not simple.

Question 4. Prove that if $|G| = 132 = 2^2 \cdot 3 \cdot 11$ then G is not simple.

Question 5. Prove that if $|G| = 231 = 3 \cdot 7 \cdot 11$ then $|Z(G)| \geq 11$ (in particular, G is not simple).

Question 6. Prove that if $|G| = 33 = 3 \cdot 11$ then G is abelian.

Question 7. If $|G| = 39 = 3 \cdot 13$, does G have to be abelian? Prove or give a counterexample.

HW 5A:	Q1, Q4, Q5
HW 5B:	Q2, Q3, Q6, Q7