Recall that a group $G$ is simple if it has no normal subgroups except itself and \{1\}.

**Question 1.** Prove that if $|G| = 312 = 2^3 \cdot 3 \cdot 13$ then $G$ is not simple.

**Question 2.** Suppose that $G$ is a simple group with $|G| = 168 = 2^3 \cdot 3 \cdot 7$. How many elements of order 7 does $G$ contain? Justify your answer.

**Question 3.** Prove that if $|G| = 56 = 2^3 \cdot 7$ then $G$ is not simple.

**Question 4.** Prove that if $|G| = 132 = 2^2 \cdot 3 \cdot 11$ then $G$ is not simple.

**Question 5.** Prove that if $|G| = 231 = 3 \cdot 7 \cdot 11$ then $|Z(G)| \geq 11$ (in particular, $G$ is not simple).

**Question 6.** Prove that if $|G| = 33 = 3 \cdot 11$ then $G$ is abelian.

**Question 7.** If $|G| = 39 = 3 \cdot 13$, does $G$ have to be abelian? Prove or give a counterexample.