Homework 1

Math 120 (Thomas Church, Spring 2018)

Due Saturday, April 7 at 11:59pm.

Do all the following exercises:

1.1.8a   1.1.9b

1.1.26

(For 1.1.8a and 1.1.9b, you can use without proof that multiplication of real or complex numbers is associative.)

Question 1. Give me an example of a group that you came up with yourself (didn't find in the textbook, or Wikipedia, etc.)

You do need to specify the operation, but you do not need to prove it is a group (although if it turns out not to be a group, we won't be able to give you many points).

You get 1 bonus point if no other student submits the same group.

Question 2. Let $G = \mathbb{R} \setminus \{0\}$ be the group of nonzero real numbers under multiplication, and let $H = \mathbb{C} \setminus \{0\}$ be the group of nonzero complex numbers under multiplication. (You don't have to prove these are groups.)

Prove that G and H are **not** isomorphic.

Question 3. We will call an element $g \in G$ an *involution* if $g^2 = 1$ but $g \neq 1$.

Let G be a finite group.

3(A) Suppose that G has an even number of elements. Show that G contains an involution.
(Hint: can you show the set $X = \{x \in G \mid x^2 \neq 1\}$ has an even number of elements?)

3(B) (Optional) Conversely, show that if G contains an involution g, then G has an even number of elements.

Question 4. (Optional) You may know that real numbers can have at most two square roots. The same is true of complex numbers. (That is, for a given b there are at most two numbers a satisfying $a^2 = b$.)

Is this true in groups? Either:

(a) Prove that for any group G and any element $b \in G$, the set $\{a \in G \mid a^2 = b\}$ of "square roots" of b contains at most two elements,

OR

(b) Give a counterexample: provide an example of a group G and an element $b \in G$ which has more than two square roots in G.

HW 1A (first PDF): 1.1.8a, 1.1.9b, Q1, Q3

HW 1B (second PDF): 1.1.26, Q2, Q4