Math 113-40, Mr. Church, Midterm Review
Wednesday, January 27

1. Give an example of a *subgroup* of S_n. Give some more.
2. In D_{10}, the group of symmetries of the pentagon, will RFR^2FR be a rotation or reflection? (Try to answer without actually finding what symmetry it is.)
3. In D_{10}, which elements commute with all other elements? (An element x commutes with all other elements if $xy = yx$ for all values of y.)
4. Recall that in a group where the identity element is called I, the *inverse* of an element f is some other element g so that $f \circ g = I$. What is the relation between the cycle decomposition of f and the cycle decomposition of g?
5. A *transposition* is cycle of length 2, a permutation which switches two numbers: for example, $(1\,3)$ is a transposition, as are $(3\,4)$, and $(2\,8)$. Prove that S_n is generated by all the transpositions.
6. Draw an octagon and consider its group of symmetries D_{16}. Find a fundamental domain and shade it in. How many translates does it have?
7. Is $(1\,2\,3\,4)(5\,6\,7)$ an *even* permutation or an *odd* permutation? How about $(1\,2)(3\,4\,5)(6\,7)$? How about [any other permutation you can think of]?
8. What is the *order* of $(1\,2)(3\,4\,5)$? How about $(1\,2\,3)(4\,5\,6)$? How about [any other permutation you can think of]?
9. If f is a permutation so that $f \circ f$ is the identity, what can you conclude about the cycle decomposition of f? Can you say anything about how many cycles f splits into? Or about the length of these cycles?
10. The Klein 4-group K_4 is the group of symmetries of a long skinny rectangle; it has four elements (the identity, the horizontal and vertical flips, and the 180-degree rotation). The cylic group $\mathbb{Z}_4$ also has four elements. So how can we be sure that these are really different groups? Find some property that K_4 has but $\mathbb{Z}_4$ could never have, no matter how you relabel the elements (or vice versa).