Question 1.

(a) Give an example of two self-adjoint operators \( S \in \mathcal{L}(\mathbb{R}^2) \) and \( T \in \mathcal{L}(\mathbb{R}^2) \) whose product \( ST \) is not self-adjoint.

Let \( V \) be a finite-dimensional inner product space, and assume that \( S \in \mathcal{L}(V) \) and \( T \in \mathcal{L}(V) \) are self-adjoint.

(b) Prove that \( ST + TS \) is a self-adjoint operator.

(c) Prove that \( ST \) is self-adjoint if and only if \( ST = TS \).