**Problem 1.** A Lie Group is a group $G$ which is also a manifold, and is such that the operations

$$G \times G \to G$$

$$(x, y) \to xy$$

and

$$G \to G$$

$$x \to x^{-1}$$

are both smooth. Show that the tangent bundle $TG$ is always trivial. That is there is an invertible map of bundles

$$TG \simeq G \times \mathbb{R}^n$$

where $n = \dim(G)$. (One example of a Lie Group is the space of invertible linear maps on finite dimensional vector space $V$.)

**Problem 2.** Remember the last definition of the tangent bundle defined its fiber $TM_p$ at a point $p \in M$ to be the vector space of derivations at $p$. That is the space of of maps:

$$C^\infty(M) \to \mathbb{R}$$

which are linear over $\mathbb{R}$ and further are “derivations at $p$”:

$$l(fg) = f(p)l(g) + g(p)l(f)$$

1. Show that the operators $l = \frac{\partial}{\partial x_i}|_p$ have this property. (partial differentiation evaluated at $p$)

2. Show that $\dim(TM_p) = \dim(M)$. For this you may find it useful to prove the following Lemma:

**Lemma 1.** Let $f$ be a $C^\infty$ function in a convex open neighborhood $U$ of $0$ in $\mathbb{R}^n$, with $f(0) = 0$. Then there are $C^\infty$ functions $g_i: U \to \mathbb{R}$ with $f(x_1, \ldots, x_n) = \sum_{i=1}^n x_i g_i(x_1, \ldots, x_n)$.

(Hint: if you have trouble, take a look at Spivak’s a Comprehensive Introduction to Differential Geometry Vol 1 pages 78-79 (in the current edition) where he discusses this definition.)

**Problem 3.** Compute the fundamental group of the complement to the Hopf link in $\mathbb{R}^3$.

**Problem 4.** Draw immersed circles in $\mathbb{R}^2$ with Whitney indices $0, 1, -1, 2$ and $-2$. 