

Math 109, Winter 2003
Monday, March 8, 2004

Instructions for Writing Assignment

GOAL: The purpose of this writing assignment is to demonstrate your understanding of group theory by showing that you can express these difficult concepts in plain English. The point is that you can't really accomplish this without fully understanding the mathematics first. In general, explaining your ideas in words in your mathematical and scientific work should force you to clarify your thinking about what you know and what you don't know.

EXPECTATION: The ultimate goal of the course is to develop skills to attack mathematical questions on your own using the logical reasoning techniques we've been practicing. Papers should possess two main components. First, they should demonstrate an understanding of the mathematical material in a well-organized and precise summary. In this portion, the best papers will attempt to re-package this information in a new way, not simply listing properties or following Armstrong's description. Second, they should address our main goal of the course, showing some evidence of mathematical analysis beyond a simple summary. This often takes the form of an exploration of questions using the tools from the course.

EXAMPLES: I hesitate to give any of these, for fear that the only papers I will receive will use these concepts alone. But to give you a better idea, here are some examples. All of these run the risk of devolving into a discussion which is not about mathematics and group theory. Make sure that your analysis uses information and logical reasoning techniques related to the course.

- (1) The Greeks believed that atomic units were made of platonic solids. Argue why this is false using a proof by contradiction based on mathematical principles of symmetry and group theory.
- (2) The Platonic solids have associated symmetric and alternating groups as their groups of symmetry. These groups have different representations (in the precise sense defined in class). What do these representations tell us about the symmetry properties of the solids.
- (3) Can we build a theory of symmetries from the Platonic solids in any way? That is, as in calculus, can we use solids to approximate other figures and infer symmetry properties of general object from those of its Platonic pieces?
- (4) For surreal numbers, you can investigate any mathematical quantities you want. This can be less about group theory and more about extending the mathematical universe described in the book. Starting from simple axioms, the characters build whole numbers. What other number systems can be constructed? Rationals? Irrationals? What advantages or disadvantages does this have over other number systems (that is, what can be computed, measured, distinguished, in one but not the other?) Do

they form a group? Under what operations? Can you construct isomorphisms or homomorphisms to other groups that help you understand the surreal numbers? What are the practical uses, if any, to developing such a number system?

ROUGH GUIDELINES: While there is no length requirement, it seems to me that a good paper would take 4 or 5 pages to explain. For the final drafts, typed papers are greatly preferred. However, they should be self-contained rather than referring to diagrams which are not included. Also remember that I am placing weight on your better performances in the class, so this is a chance to do something good to give me something to think about when preparing grades, especially if you haven't performed as well as you would like on the midterms.