

MATH 52 FINAL EXAM

1. (a) Sketch the region R of integration in the following double integral.

$$\int_0^1 \int_{\sqrt{x}}^1 x e^{y^5} dy dx$$

- (b) Express the region R as an x -simple region.
 (c) Evaluate the integral by changing the order of integration.
2. (a) Let T be a solid cone whose base is the disk $x^2 + y^2 \leq 1$ in the xy -plane and whose vertex is the point $(0, 0, 2)$. Set up, but do not evaluate, a triple integral in cylindrical coordinates which computes the moment of inertia of T around the x -axis.
 (b) Let T be the solid ball of radius 1 centered at $(2, 0, 0)$. Set up, but do not evaluate, a triple integral in spherical coordinates which computes the moment of inertia of T around the z -axis. **Hint:** First consider the change of variables $u = x - 2$, $v = y$, $w = z$.

3. Consider the curve C parametrized by $\mathbf{r}(t) = (2e^t, \frac{1}{3}e^{3t} + e^{-t})$ for $0 \leq t \leq 1$.

- (a) Find the arclength of C .
 (b) Find the x -coordinate of the centroid of C .
 (c) Let S be the surface obtained by rotating C around the line $x = -1$. Find the area of S .
4. (a) Consider the change of variables $u = \frac{x^2}{4} + y^2$, $v = \frac{y}{x}$. Find the Jacobian

$$\frac{\partial(x, y)}{\partial(u, v)}$$

- (b) Find the area of the region in the first quadrant enclosed by the ellipses $\frac{x^2}{4} + y^2 = 1$, $\frac{x^2}{4} + y^2 = 4$ and the lines $y = 2x$ and $y = 4x$.
5. Let T be the solid enclosed by the planes $z = x + y$, $x + 2y = 2$ and the paraboloid $z = y^2$. Set up, but do not evaluate a triple integral in rectangular coordinates which computes the volume of T by
- (a) regarding T as x -simple.
 (b) regarding T as z -simple.
6. Let C be the curve parametrized by $\mathbf{r}(t) = (t^2, \sin t)$ for $0 \leq t \leq \pi/2$.

- (a) Compute $\int_C y dx$.

- (b) Let R be the region in the plane bounded by the y -axis, the line $y = 1$, and the curve C . Use Green's Theorem to find the area of R .
7. Let S be the portion of the cylinder $x^2 + y^2 = 2x$ which lies above the xy -plane and below the surface $z = x^2$.
- (a) Write down a parametrization of S . Be sure to specify the domain.
- (b) Find the area of S .
8. Let S be the surface parametrized by $\mathbf{r}(u, v) = (v^2, u^2, 2uv)$ over the domain $D = [-1, 1] \times [-1, 1]$ in the uv -plane.
- (a) Find the area of S .
- (b) Find the x -coordinate of the centroid of S .
9. Let S be the portion of the sphere $x^2 + y^2 + z^2 = 4$ which lies above the plane $z = -1$, and let $\mathbf{F}(x, y, z) = (x - 3y + 2z, -3x + 2y + z, 2x + y - 3z)$. Compute the flux

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS,$$

where \mathbf{n} denotes the upward unit normal to S .

10. Let C be the triangle with vertices $A = (1, 1, 1)$, $B = (3, -1, 0)$ and $C = (1, -1, 2)$, parametrized in that order. Let $\mathbf{F}(x, y, z) = (4z^2, 2x^2, y^2)$.
- (a) Find the area of the triangle.
- (b) Find the equation of the plane in which the triangle lies.
- (c) Compute $\text{curl } \mathbf{F}$.
- (d) Compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$.
11. Let $\mathbf{F}(x, y, z) = (2xy + z^2, 2yz + x^2, 2zx + y^2)$.
- (a) Find a potential for \mathbf{F} .
- (b) Let C be the curve parametrized by $\mathbf{r}(t) = (\cos t, \sin t, t)$ for $0 \leq t \leq \pi$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$
12. Let T be the solid region which lies below the surfaces $z = 1 - x^2$ and $z = 1 - y^2$ and above the xy -plane.
- (a) Find the volume of T .
- (b) Find the outward flux through the boundary of T of the vector field $\mathbf{F}(x, y, z) = (zx^2, z^2 - y, 3z - z^2x)$.