Publications

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Publications

(Preprints of all submitted papers are available at `math.stanford.edu/~brubaker`)

1. **Non-vanishing twists of GL(2) automorphic L-functions**, with A. Bucur, G. Chinta, S. Frechette, and J. Hoffstein, *Int. Math. Res. Not.* (2004) **78**, 4211–4239.

   Let π be a cuspidal automorphic representation of $GL(2, \mathbb{A}_K)$, K a number field containing the pth roots of unity. Given a prime p, suppose there exists a single non-vanishing p^{th} order twist of the L-series associated to π at the center of the critical strip. We use the method of multiple Dirichlet series to establish that there exist infinitely many such non-vanishing p^{th} order twists of the L-series of the representation at the center.

2. **Cubic twists of GL(2) automorphic L-functions**, with S. Friedberg and J. Hoffstein, *Invent. Math.* (2005) **160**, no. 1, 31–58.

   Let $K = \mathbb{Q}(\sqrt{-3})$ and let π be a cuspidal automorphic representation of $GL(2, \mathbb{A}_K)$. Consider the family of twisted L-functions $L(s, \pi \otimes \chi)$ where χ ranges over the cubic Hecke characters of K. In this paper the mean value of this family of L-functions is computed; the result is consistent with the generalized Lindelöf hypothesis. From this mean value result a nonvanishing theorem is established: for given s there are infinitely many cubic twists such that the L-value at s is nonzero. At the center of the critical strip the number of such characters of norm less than X is $\gg X^{1/2-\epsilon}$. These results are obtained by introducing and studying three different families of weighted double Dirichlet series. These series are related by functional equations, some of which are obtained through the study of higher metaplectic Eisenstein series and the Hasse-Davenport relation. The authors establish the continuation of such series and then obtain their main result by Tauberian methods.

3. **On Kubota's Dirichlet series**, with D. Bump. To appear in *J. Reine Angew. Math.*

   We exhibit concrete finite-dimensional families of Dirichlet series that are closed under certain functional equations. The description of these Dirichlet series is as

simple as we can imagine. They have the form

$$\mathcal{D}(s,\Psi,\alpha)=\sum_{0\neq c\in\mathfrak{o}_S/\mathfrak{o}_S^\times} g(\alpha,c)\,\Psi(c)\,\mathbb{N}(c)^{-2s}\,, \tag{1}$$

where $\mathfrak{o}_S$ is the ring of S-integers in a number field, $g(\alpha,c)$ a Gauss sum and the function Ψ is restricted to a finite-dimensional vector space of functions making the Dirichlet series well defined if c is replaced by εc, where $\varepsilon\in\mathfrak{o}_S^\times$. We bound the finite Dirichlet polynomials that appear in the functional equation uniformly in α and s, required for applications. We also prove an equivariance of these polynomials under a natural action of $\mathfrak{o}_S^\times$, also needed for applications.

4. **Weyl group multiple Dirichlet series I**, with D. Bump, G. Chinta, S. Friedberg, and J. Hoffstein. Submitted for publication.

   We introduce a family of Dirichlet series which conjecturally satisfy a group of functional equations isomorphic to the Weyl group. Emphasis is placed on heuristic methods relating these series to Dynkin diagrams, connections with metaplectic Eisenstein series, and a careful proof is given in the case $\Phi=A_2$. Complete proofs of these conjectures are offered in the following paper.

5. **Weyl group multiple Dirichlet series II**, with D. Bump and S. Friedberg. To appear in *Invent. Math.*

   To each reduced root system Φ of rank r, and each sufficiently large integer n, we define a family of multiple Dirichlet series in r variables, whose group of functional equations is isomorphic to the Weyl group of Φ. The coefficients in these Dirichlet series exhibit a multiplicativity that reduces the specification of the coefficients to those that are powers of a single prime p. For each p, the number of nonzero such coefficients is equal to the order of the Weyl group, and each nonzero coefficient is a product of n-th order Gauss sums. The root system plays a basic role in the combinatorics underlying the proof of the functional equations.

6. **Weyl group multiple Dirichlet series III**, with D. Bump, S. Friedberg, and J. Hoffstein. Submitted for publication.

   There are three main purposes of this paper. The first is to give evidence for the connection between Weyl group multiple Dirichlet series and Eisenstein series by explicitly computing the $1,1$-Whittaker coefficient of a metaplectic GL_3 Eisenstein series, and identifying it with a certain multiple Dirichlet series $Z(s_1,s_2)$ when the root system $\Phi=A_2$. The second purpose is to compute the m_1,m_2-Whittaker coefficient of the same Eisenstein series, and to infer from it the correct definition for a generalization $Z(s_1,s_2;m_1,m_2)$ that agrees with $Z(s_1,s_2)$ when $m_1=m_2=1$. Finally, We give a description of the A_r Weyl group multiple Dirichlet series in the unstable case (that is, without restriction on degree of the cover). The p-part of the multiple Dirichlet series is given as a finite sum of products of Gauss sums

parametrized by Gelfand-Tsetlin patterns. This description is proved for $r = 2$, and is conjectural for $r > 2$, but supported by strong evidence.

7. **Residues of Weyl Group Multiple Dirichlet Series Associated to GL(n)**, with D. Bump. Submitted for publication.

   We compute residues of a Dirichlet series in three complex variables associated to the root system A_3 and show that it explains hidden functional equations in a Dirichlet series first investigated by Friedberg, Hoffstein, and Lieman in order to obtain mean values for Dirichlet L-functions. We show the case of cubic Dirichlet L-series explicitly and the make conjectures about how to obtain n-th order twists using residues of the A_n diagram.

8. **Mean values for cubic twists of Dirichlet L-functions**, In preparation.

   The main result is determination of exact functional equations which provide the analytic continuation of a Dirichlet series whose numerator consists of the product of two cubic Dirichlet L-series. As an application, the first asymptotics for the second moment of cubic Dirichlet L-series are obtained via Tauberian techniques. This is a work in progress to expand the results of my Ph.D. thesis to include the new perspective gained by associating the Dirichlet series to a residue of an Eisenstein series associated to the exceptional group E_6.

Collaborators and Other Affiliations

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