Math 109, Winter 2003

Inaugural Proof and Pizza Workshop

Wednesday, January 21, 2004

(1) If G is abelian, prove that $(a * b)^n = a^n * b^n$ for all integers n.

(2) Show that any group of order 5 must be abelian.

(3) If G is a finite group, show that given any $a \in G$ there is a positive integer n depending on a such that $a^n = e$.

(4) Let G be a finite set with operation $*$ such that

   (a) G is closed under $*$.

   (b) $*$ is associative.

   (c) Given $a, b, c \in G$ with $a * b = a * c$, then $b = c$.

   (d) Given $a, b, c \in G$ with $b * a = c * a$, then $b = c$.

   Prove that G must be a group under $*$.

(5) If G has no proper subgroups, prove that G is cyclic.

(6) Prove that for any k, the composition of transpositions in S_k satisfies the following equality:

$$(1\ k)(1\ (k-1)) \cdots (1\ 3)(1\ 2) = (1\ 2\ 3 \cdots k)$$

(7) If A, B are subgroups of an abelian group G, let $AB = \{a * b \text{ such that } a \in A, b \in B\}$. Prove that AB is a subgroup of G. Prove or Disprove: This still holds if G is not abelian.

(8) Show that $\mathbb{R}$ is not a cyclic group.