

MATH 52 SAMPLE MIDTERM II

November 13, 2004

Name: _____

Numeric Student ID: _____

I agree to abide by the terms of the honor code:

Signature: _____

Instructions: Print your name, and student ID number in the space provided. During the test you may not use notes, books, or calculators. Read each question carefully. State clearly any facts you use from class or the textbook. Correct answers without justification will receive little or no credit. There are 5 questions. You have 120 minutes to do all the problems.

Question	Score	Maximum
1		20
2		10
3		10
4		10
5		10
Total		60

1. Evaluate the following line integrals:

(a)

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $\mathbf{F}(x, y, z) = \langle xy, yz, xz \rangle$ and $C : \mathbf{r}(u) = \langle u, u^2, u^3 \rangle$ from $(-1, 1, -1)$ to $(1, 1, 1)$.

(b)

$$\int_C \mathbf{F} \cdot \mathbf{T} ds$$

where $\mathbf{F}(x, y, z) = \langle \cos x, \sin y, yz \rangle$ and C is the line segment from $(0, 0, 0)$ to $(2, 3, -1)$.

(c) Compute the work done by

$$\mathbf{F}(x, y, z) = \left\langle 2x \ln y - yz, \frac{x^2}{y} - xz, -xy \right\rangle$$

along the line segment from $(1, 2, 1)$ to $(3, 2, 2)$.

(d) Evaluate

$$\int_C \mathbf{F} \cdot \mathbf{T} ds$$

where $\mathbf{F}(x, y) = \langle 3xy + y^2, 2xy + 5x^2 \rangle$ and $C : (x - 1)^2 + (y + 2)^2 = 1$ is traversed in the counter-clockwise direction.

2. Compute the value of the surface integral

$$\iint_S (x^2 + y^2) dS$$

where S is the surface $z = 1 - x^2 - y^2$ above the coordinate plane.

3. Verify the divergence theorem for the vector field $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$ and the closed surface S , a sphere of radius a centered at the origin. That is, calculate both the flux out of the surface and the integral of the divergence and note they are equal.

4. Compute the line integral

$$\oint_C \frac{x}{x^2 + y^2} dx + \frac{y}{x^2 + y^2} dy$$

over any closed curve C.

5. The Folium of Descartes is the curve given by

$$x^3 + y^3 = 3axy, \quad a > 0 \text{ some fixed constant.}$$

If we let $t = y/x$ then the folium can be parametrized by the following set of equations:

$$x = \frac{3at}{1+t^3} \quad y = \frac{3at^2}{1+t^3} \quad t \neq -1$$

- (a) Graph the folium by plotting a few points.

- (b) Use Green's theorem to find the area of the closed loop.

- (c) Compute an improper integral to show that the area of the region between the asymptote $x + y + a = 0$ and the folium equals the area of the closed loop.