$\boxed{3.5}$ (c) Assume $h(x,y)\,k(\cancel{z},w) = G(B_1(x,y;z,w), B_2(x,y;z,w))$

is a direct composition, in the sense of 3.1.

$\tilde{h} \sim h$; namely $\tilde{h}(x,y) = h(px+qy, rx+sy)$ for $ps-qr=1$.

Then we obviously have:

$$\tilde{h}(x,y)\,k(z,w) = G(B_1(px+qy, rx+sy; z,w), B_2(px+qy, rx+sy; z,w))$$

We want to prove this is direct composition in the sense of 3.1, namely we want to check that:

(i) $k(1,0) = B_1(p,r;1,0)\,B_2(q,s;1,0) - B_2(p,r;1,0)\,B_1(q,s;1,0)$

But $RHS = (a_1p+c_1r)(a_2q+c_2s) - (a_2p+c_2r)(a_1q+c_1s)$

$= a_1c_2(ps-qr) - a_2c_1(ps-qr) = a_1c_2 - a_2c_1 = k(1,0)$, as desired

and

(ii) $\tilde{h}(1,0) = h(p,r) \overset{?}{=} B_1(p,r;1,0)\,B_2(p,r;0,1) - B_1(p,r;0,1)\,B_2(p,r;1,0)$

$= (a_1p+c_1r)(b_2p+d_2r) - (a_2p+c_2r)(b_1p+d_1r)$

Since both sides are binary quadratic forms in variables p and r, it suffices to check:

$p=1$ $q=0$ $h(1,0) = a_1b_2 - a_2b_1$ ✓ (Already assumed)

$p=0$ $q=1$ $h(0,1) = c_1d_2 - c_2d_1$

$p=1$ $q=1$ $h(1,1) = (a_1+c_1)(b_2+d_2) - (a_2+c_2)(b_1+d_1)$.

And these are proved following Exercise 3.1.