$$\Leftrightarrow \underbrace{x^2+ny^2}_{\text{opposite of itself}} \sim \underbrace{px^2-bxy+cy^2}_{\text{opposite of } px^2+bxy+cy^2} \quad \Big\} \quad x^2+ny^2 \sim \underbrace{px^2 \pm bxy + c^2}_{g}$$

$\Rightarrow g = x^2+ny^2$, as it is reduced and <u>properly</u> equiv. to x^2+ny^2.

$\boxed{3.5}$ (a) Choose $q \in \mathbb{Z}$ so that $B-b = 2a\cdot q$.

Then $a(x+qy)^2 + b(x+qy)y + cy^2 = ax^2 + Bxy + \tilde{C}y^2$.

But also $B^2 - 4a\tilde{C} = D \Rightarrow \tilde{C} = \dfrac{B^2-D}{4a} = a'C$.

So we proved $ax^2+bxy+cy^2 \sim ax^2+Bxy+a'Cy^2$, as desired.

(b) Notice that

$$\left(ax + \frac{B+\sqrt{D}}{2}y\right)\left(ax + \frac{B-\sqrt{D}}{2}y\right) = a\cdot f(x,y)$$

To see this, try to factor the quadratic $f(x,y)$; namely find its homog. solutions

$$\left(a'z + \frac{B+\sqrt{D}}{2}w\right)\left(a'z + \frac{B-\sqrt{D}}{2}w\right) = a'\cdot g(x,y) \quad (*)$$

Multiplying both sides we see that

$$aa'f(x,y)\,g(x,y) = \left(ax + \frac{B+\sqrt{D}}{2}y\right)\left(a'z + \frac{B+\sqrt{D}}{2}w\right)\left(ax + \frac{B-\sqrt{D}}{2}y\right)\left(a'z + \frac{B-\sqrt{D}}{2}w\right)$$

$$\underset{\text{Hint and } (*)}{=} F(x,y)$$

To conclude that this is direct composition we <u>need to check</u>:

$\boxed{a_1b_2 - a_2b_1 = a, \quad a_1c_2 - a_2c_1 = a'}$; where $a_1, b_1, a_2, b_2, c_1, c_2$ are as in 3.1.

But by definition, our $a_1 = 1$, $d_1 = -c$, $b_1 = 0$, $c_1 = 0$, $a_2 = 0$, $b_2 = a$, $c_2 = a'$, $d_2 = B$. From this we easily verify the identities above. $\square$

(3)