

2.21 By 2.19 above, the principal genus consists only of the principal reduced form $x^2 + by^2$ (for $D = -24$); and by Corollary 27; p is represented by (a/the) form in the principal genus $\Leftrightarrow p \equiv \beta^2$ or $\beta^2 + 6 \pmod{24} \Leftrightarrow p \equiv 1, 7 \pmod{24}$.
 (The rest of the statements are proved exactly the same way)

2.27 (a) Using the hint, we see that (\sim : "proper" equivalence)
 $f \sim px^2 + bxy + cy^2$
 $g \sim px^2 + \beta xy + \theta y^2$
 for some $b, c, \beta, \theta \in \mathbb{Z}$.

By the reduction theorem, we see that we may assume $|b| \leq p$, $|\beta| \leq p$.
 (PROOF of the)

But since $\text{discr}(f) = \text{discr}(g)$, it follows that

$$(*) \quad b^2 - 4pc = \beta^2 - 4p\theta \quad \Rightarrow \quad b \equiv \mp \beta \pmod{p} \quad (\text{only if } \begin{matrix} b > 0 \\ b < 0 \end{matrix})$$

$$\Rightarrow \quad b \equiv \beta \pmod{2}$$

But since $b, \beta \in [-p, p]$, we see that $\beta \in \{b, -b, \begin{matrix} p+b \\ p-b \end{matrix} \}$ (if $b < 0$)

But since $b \equiv \beta \pmod{2}$, it follows that $\beta \in \{b, -b\}$ is the only possibility, hence by (*) $c = \theta$ as well.

So $f \sim px^2 + bxy + cy^2$
 $g \sim px^2 \pm bxy + cy^2$ } These two are (possibly improperly) equivalent. \square

(b) By the proof of part (a) above, we see that

$$x^2 + ny^2 \sim px^2 + bxy + cy^2$$

$$g(x, y) \sim px^2 \pm bxy + cy^2$$

② By problem 2.25 (or 2.26) $x^2 + ny^2 \sim px^2 + bxy + cy^2 \Leftrightarrow$ Their opposites are equiv. \Leftrightarrow