

2.18 a) If $f(x,y) = ax^2 + bxy + cy^2$, then $f(1,0) = a$, $f(0,1) = c$

$f(1,1) = a + b + c$. If $p|a$, $p|c$, $p|a+b+c \Rightarrow p|b$ as well, contradicting the assumption that f is primitive.

b) For any $p|M$, choose $\alpha_p \in \{0,1\}$ and $\beta_p \in \{0,1\}$ so that

$$\gcd(f(\alpha_p, \beta_p), p) = 1.$$

Now, by Chinese Remainder theorem, there is a α , and a β s.t.

$$\alpha \equiv \alpha_p \pmod{p} \quad \forall p|M$$

$$\beta \equiv \beta_p \pmod{p} \quad \forall p|M$$

Then one easily checks $\gcd(f(\alpha, \beta), M) = 1$.

2.19 First we list all the reduced forms of disc. $D = -15, -24, -31, -52$.

$D = -15$: $x^2 + xy + 4y^2$ represents $\{1, 4\}$ in $(\mathbb{Z}/15\mathbb{Z})^\times$ (Two genera)
 $2x^2 + xy + 2y^2$ " $\{2, 8\}$ in $(\mathbb{Z}/15\mathbb{Z})^\times$

$D = -24$: $x^2 + 6y^2$ represents $\{1, 7\}$ in $(\mathbb{Z}/24\mathbb{Z})^\times$ (Two genera)
 $2x^2 + 3y^2$ " $\{5, 11\}$ in $(\mathbb{Z}/24\mathbb{Z})^\times$

$D = -31$: $x^2 + xy + 8y^2$ There is only one genus.
 $2x^2 \pm xy + 4y^2$ (For this, one might find useful to utilize some genus theory, which gives this answer straight away)

$D = -52$: $x^2 + 13y^2$ represents $\{1, 9, 25, 27, 29, 49\}$ in $(\mathbb{Z}/52\mathbb{Z})^\times$
 $2x^2 + 2xy + 7y^2$ " $\{7, 11, 17, 19, 31, 47\}$ " (Two genera)