

Now following the arguments for (2.20), we see that all the forms $2x^2 + 7y^2$, $3x^2 \pm 2xy + 5y^2$ (the ^{rest of the} reduced forms of disc. -56) represent exactly six elements; and since $3x^2 \pm 2xy + 5y^2$ both represent 3, the set of elements represented by these two forms should be the complement of the set represented by $x^2 + 14y^2$ (by Thm. 2.16). Hence, $3x^2 \pm 2xy + 5y^2$ represents only $3, 5, 13, 19, 27, 45 \pmod{56}$ and $x^2 + 14y^2$ and $2x^2 + 7y^2$ — only $1, 9, 15, 23, 25, 39 \pmod{56}$.

(2.15) $p \equiv \ker \chi = \{1, 9, 15, 23, 25, 39, 3, 5, 13, 19, 27, 45\} \pmod{56}$ is represented by a \downarrow form of disc. -56 , so by (reduced)

one of $x^2 + 14y^2$, $2x^2 + 7y^2$ or $3x^2 \pm 2xy + 5y^2$.

But 2.21 tells us that $x^2 + 14y^2$, $2x^2 + 7y^2$ cannot represent primes $p \equiv \{3, 5, 13, 19, 27, 45\}$; and $3x^2 \pm 2xy + 5y^2$ cannot represent primes $p \equiv \{1, 9, 15, 23, 25, 39\}$; which in return implies

$x^2 + 14y^2$ or $2x^2 + 7y^2$ represent a prime $p \Leftrightarrow p \equiv 1, 9, 15, 23, 25, 39 \pmod{56}$
 $3x^2 \pm 2xy + 5y^2$ ————— $\Leftrightarrow p \equiv 3, 5, 13, 19, 27, 45 \pmod{56}$