In fact the reduced forms of discr $= -124$ are
x^2+31y^2, $5x^2 \pm 4xy+7y^2$.

(2.14) Proof of (2.20):

Letting $x=1$, $y=0$, $x^2+5y^2=1$
$x=2$ $y=1$, $x^2+5y^2=9$

Therefore x^2+5y^2 represents 1 and 9 mod 20.

Further, it does NOT represent 3:
If $x^2+5y^2 \equiv 3 \pmod{20} \Rightarrow x^2+y^2 \equiv 3 \pmod 4$ which is impossible as $x^2, y^2 \equiv 0$ or $1 \pmod 4$.

But since the set of elements that the principal reduced form x^2+5y^2 represents is a <u>subgroup</u> of $\{1,3,7,9\} = \ker \chi$ it follows that this set should have exactly 2-elements, which means x^2+5y^2 represents <u>only</u> 1 and 9 mod 20.

Now $2x^2+2xy+3y^2$ is <u>the</u> only non-principal reduced form with discriminant -20, and by Lemma 2.5, we know that it should represent 3 and 7 mod 20. Further, by Thm 2.16, the set of elements that $2x^2+2xy+3y^2$ represents has the same number of elements as the set of elts that the principal reduced form represents. So 2.20 is proved.

Proof of (2.21):

One easily verifies that both x^2+14y^2 and $2x^2+7y^2$ represents $1, 9, 15, 23, 25, 39 \pmod{56}$. Once again, we prove that x^2+14y^2 does not represent $3 \pmod{56}$ (from which it will follow that (using Thm 2.16) x^2+14y^2 represents exactly the above 6 elements; recalling that $\ker \chi$ has 12 elements):

If $x^2+14y^2 \equiv 3 \pmod{56} \Rightarrow x^2+6y^2 \equiv 3 \pmod 8$ But $x^2 \equiv 0,1,4 \pmod 8$, $6y^2 \equiv 0,6 \pmod 8 \Rightarrow x^2+6y^2 \equiv 0,1,4,6,7,2 \pmod 8$, not 3; so x^2+14y^2 cannot represent $3 \pmod{56}$)