2.11 By Lemma 2.5 p is represented by a prim. form of discr. $D=-4n$ (for $n=1,2,3,7$) iff $p=D$ or $\left(\frac{D}{p}\right)=\left(\frac{-n}{p}\right)=1$. By Table 2.14, there is only one reduced form of disc $=D$ (for D as above), namely, x^2+ny^2, which means

(*) $x^2+ny^2=p \iff \left(\frac{-n}{p}\right)=1$ or $p=n$ (for $n=1,2,3,7$).

But $\left(\frac{-1}{p}\right)=\begin{cases}1 & p\equiv 1\ (4)\\ -1 & p\equiv 3\ (4)\end{cases}$ $\quad \left(\frac{-2}{p}\right)=\begin{cases}1 & p\equiv 1,3\ (8)\\ -1 & p\equiv 5,7\ (8)\end{cases}$

$\left(\frac{-3}{p}\right)=\left(\frac{p}{3}\right)=\begin{cases}1 & p\equiv 1\ (3)\\ -1 & p\equiv 2\ (3)\end{cases}$ $\quad \left(\frac{-7}{p}\right)=\left(\frac{p}{7}\right)=\begin{cases}1 & p\equiv 1,2,4\ (7)\\ -1 & p\equiv 3,5,6\ (7)\end{cases}$

Combine these arguments with (*) to conclude.

2.12 b Assume $f(x,y)=ax^2+bxy+cy^2$ is a reduced form of discr. D.

(i) $D=-32$

Then $a<\sqrt{\frac{32}{3}}$, so $a\le 3$; and since $b^2-4ac=-32$, b is even. So we have only the following cases:

$a=1 \Rightarrow b=0,\ c=8$ (get x^2+8y^2)

$a=2 \Rightarrow b=0$ or ∓ 2, and $c=4$ or $9/2$, and either of them are impossible (since $\gcd(2,0,4)=2\neq 1$)

$a=3 \Rightarrow b=0$ or ∓ 2, and $c=8/3$ or 3

Get $3x^2+2xy+3y^2$ (notice that $3x^2-2xy+3y^2 \sim 3x^2+2xy+3y^2$ and only latter is reduced, by definition)

So $h(D)=2$ in this case.

(ii) $D=-124$

Following the arguments above, only the following cases are possible:

$a=1 \Rightarrow b=0,\ c=31$ (get x^2+31y^2)

$a=2 \Rightarrow b=0$ or ∓ 2, $c=\frac{31}{2}$ or $c=16$ (impossible)

$a=3 \Rightarrow b=0$ or $b=\mp 2$, $c=\frac{31}{3}$ or $\frac{32}{3}$ (impossible)

$a=4 \Rightarrow b=0$ or $b=\mp 2$ or $b=\mp 4$, $c=\frac{31}{4}$ or $c=8$ or $\frac{35}{4}$ (impossible)

$a=5 \Rightarrow b=0$ or $b=\mp 2$ or $b=\mp 4$, $c=\frac{31}{5}$ or $\frac{32}{5}$ or 7

Get $5x^2\pm 4xy+7y^2$ $\Rightarrow h(D)=3$ in this case.