(2.6) A form (positive definite) $f(x,y) = ax^2 + bxy + cy^2$ is reduced if $-a < b \leq a < c$

or $0 \leq b \leq a = c$

In our case $f(x,y) = 126x^2 + 74xy + 13y^2$.

$f \sim g$ where $g(x,y) = f(-y,x) = 126y^2 - 74xy + 13x^2$ and

$g \sim h$ where $h(x,y) = g(x+3y, y) = 13x^2 + 4xy + 243y^2$

and obviously h is reduced.

(2.10) (a) Among all forms properly equivalent to a given one, choose one – say f – with $|b|$ minimal. Assume $|b| > |a|$.

Then $f(x+my, y) \sim f$

$$\| $$

$ax^2 + (2am+b)xy + c'y^2$. But when $|b| > |a|$, we can choose m properly so that $|2am+b| < |a| < |b|$, which would contradict the choice of f. This proves $|a| \geq |b|$. Similarly, one proves $|c| \geq |b|$.

Now if $|c| \geq |a|$, then we already have $|c| \geq |a| \geq |b|$, therefore we're done.

If $|c| < |a|$, then $f(-y,x) \sim f(x,y)$ and the coefficients of $f(-y,x) = ay^2 - bxy + cx^2$ satisfy the desired inequality.

(b) Assume contrary that $|a| > \frac{\sqrt{D}}{2}$, then $|c| > \frac{\sqrt{D}}{2}$ as well, so $|ac| > \frac{D}{4}$, hence $4|ac| > D$. This proves that, since $b^2 - 4ac = D$, a and c have the same sign; say wlog $a, c > 0$.

This means $b^2 > 2D$, therefore $|b| > \sqrt{2D}$, hence $|c| \geq |a| > \sqrt{2D}$, hence $4ac = 4|ac| > 8D \Rightarrow |b| > 3\sqrt{D}$. Using this argument over and over again, we prove $|b|$ is infact bigger than any positive number, which is absurd. Therefore $|a| < \frac{\sqrt{D}}{2}$.

(c) By (a) and (b), any form is equivalent to a form $ax^2 + bxy + cy^2$ $|b| \leq |a| < |c|$ with $|a| \leq \frac{\sqrt{D}}{2}$, so there are finitely many choices for a, b and $\frac{b^2 - D}{4a} = c$