$$p \mid N_0/q = z^2+2w^2$$
$$(z,w)=1$$

This time contradicting the minimality of N_0.

This proves that such a p should not exist. $\square$

(b) ($n=3$ case)

Assume as above p is the smallest odd prime s.t.

$p \mid a^2+3b^2$, with $(a,b)=1$; but $p \neq x^2+3y^2$ for any $x,y \in \mathbb{Z}$.

And similarly (for fixed p as above) let $N_0 = a_0^2+3b_0^2$ be the smallest integer s.t.

$p \mid N_0$, and $(a_0,b_0)=1$.

<u>Claim 1</u>: $N_0 < p^2$

<u>Proof</u>: identical to <u>claim 1</u> in part (a).

<u>Claim 2</u>: N_0 is odd.

<u>Proof</u>: Assume $2 \mid N_0 = a_0^2+3b_0^2$. Then one checks easily that $4 \mid a_0^2+3b_0^2$; since $4 = 1^2+3.1^2$, using problem 2(b) it follows that $N_0/4 = z^2+3w^2$, $(z,w)=1$; and $p \mid N_0/4$. This contradicts the minimality of N_0; therefore $2 \nmid N_0$.

Now the rest of the proof is identical to part (a). $\square$