PART I

1) a) $(x^2+ny^2)(z^2+nw^2) = x^2z^2 + n^2y^2w^2 + ny^2z^2 + nx^2w^2 =$

$$= x^2z^2 + n^2y^2z^2 \pm 2nxyzw + n(y^2z^2 + x^2w^2 \mp 2xyzw) =$$

$$= (xz \pm nyz)^2 + n(yz \mp xw)^2. \qquad \square$$

b) $(ax^2+cy^2)(az^2+cw^2) = (axz \mp cyw)^2 + ac(xw \pm yz)^2.$

2) a) Claim 1: $q \nmid b$.

Proof: Assume contrary. Then since $q \mid N$, $q \mid N - nb^2 = a^2 \Rightarrow$

$q \mid a^2 \underset{q \text{ is prime}}{\Rightarrow} q \mid a$; contradiction.

Claim 2: $a^2y^2 \equiv b^2x^2 \pmod{q}$, therefore $ay \equiv \mp bx \pmod{q}$.

Proof: $N = a^2 + nb^2 \equiv 0 \pmod{q} \Leftrightarrow n \equiv -a^2b^{-2} \pmod{q}$

(This operation is valid since $(b,q)=1$)

But also $q = x^2 + ny^2 \equiv 0 \pmod{q} \Rightarrow x^2 - a^2b^{-2}y^2 \equiv 0 \pmod{q}$

$\Leftrightarrow a^2y^2 \equiv b^2x^2 \pmod{q}$. $\square$

Now notice that $q = x^2+ny^2 = (-x)^2 + ny^2$, so replacing x by $(-x)$ if neccassary, we may assume wlog that (and using Claim 2)

$$ay \equiv bx \pmod{q}.$$