a) $9^{72} \equiv 1 \pmod{73} \Rightarrow 9^{794} \equiv 9^2 \pmod{73} \Rightarrow a = 8$ is a solution

b) $x^{86} \equiv x^2 \pmod{29}$ so $x^{86} \equiv 6 \pmod{29} \Leftrightarrow x^2 \equiv 6 \pmod{29}$

$x = 8$ and $x = 21$ are the only solutions to the latter equatio

c) $x^{39} \equiv x^3 \pmod{13}$ so $x^{39} \equiv 6 \pmod{13} \Leftrightarrow x^3 \equiv 6 \pmod{13}$

Latter equation has no solutions. (Here is one way to see this:

$6^4 \equiv 36^2 \equiv 10^2 \equiv 9 \pmod{13}$, so if $x^3 \equiv 6 \pmod{13}$, we would h

$x^{12} \equiv 6^4 \equiv 9 \pmod{13}$, contrary to Fermat's Little Theorem)

2-

$(p-1)! \equiv (p-1) \pmod{p}$

PROOF: Let $x \in \{2, \dots, p-2\}$ (so $x \leq p-1$, but we exc 1 & $p-1$ for now) Then $\exists!\, y_x \in \{2, \dots, p-2\}$ such that $x \cdot y \equiv 1 \pmod{}$

Further $x \neq y_x$; in fact if $x = y_x$, then we would have

$x^2 = xy_x \equiv 1 \pmod{p} \Leftrightarrow (x-1)(x+1) \equiv 0 \pmod{p} \Leftrightarrow x \equiv 1$ or $x \equiv p-1$

So $(p-1)! = 1 \cdot (p-1) \cdot \prod_{\substack{x \in \{2, \dots, p-2\} \\ x < y_x}} x \cdot y_x \equiv (p-1) \pmod{p}$. $\square$

3-

$ac \equiv bc \pmod{m} \Leftrightarrow m \mid ac - bc = c(a-b) \underset{(m,c)=1}{\Leftrightarrow} m \mid a-b \Leftrightarrow$

$\Leftrightarrow a \equiv b \pmod{m}$