MATH 109 SAMPLE MIDTERM II

Wednesday, February 23, 2005

Name: ____________________________________

Numeric Student ID: ________________________

I agree to abide by the terms of the honor code:

Signature: _________________________________

Instructions: Print your name, and student ID number in the space provided. You may not use notes or any text. Read each question carefully. Some questions continue onto a second page. Try to be as precise as possible when writing your proofs. Correct answers without justification will receive little or no credit. There are 5 questions. You have 90 minutes to complete all the problems.

Question	Score	Maximum
1		15
2		10
3		15
4		10
5		10
Total		60

 Name: ______________________________

1. Provide a brief definition AND an example of the following concepts:

   (a) An equivalence relation $\sim$ on a set X.

   (b) A normal subgroup H in a group G.

   (c) The center $Z(G)$ of a group G.

   (d) The commutator subgroup $[G, G]$ of a group G.

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2. Let H be a subgroup of G. Do sets of the form

$$S = \{y^{-1}x \mid x, y \in G, y^{-1}x \in H\}$$

partition G? Why or why not?

3. Separate the following groups into isomorphism classes. That is, determine which groups, if any, are isomorphic to each other in the following list:

$$\mathbb{Z}/16\mathbb{Z}, \qquad \mathbb{Z}/8\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}, \qquad D_8, \qquad D_4 \times \mathbb{Z}/2\mathbb{Z}, \qquad A_4$$

Be sure to explain your reasoning.

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4. (a) Prove or Disprove: The group G_n defined for any integer n by

$$G_n = S_n/A_n \times D_n/\langle r \rangle \times \mathbb{Z}/7\mathbb{Z}$$

is abelian for any choice of n. Here $\langle r \rangle$ denotes the subgroup generated by a rotation r such that $r^n = e$.

(b) Prove or Disprove that the group G_n defined in part (a) is cyclic.

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5. Prove that all groups of order $2p$ are either cyclic or dihedral. That is, show that any such group G is isomorphic to C_{2p}, the cyclic group with $2p$ elements, or D_p, the group of symmetries of a p-gon.