

MATH 109 SAMPLE MIDTERM

Friday, January 21, 2005

Name: _____

Numeric Student ID: _____

I agree to abide by the terms of the honor code:

Signature: _____

Instructions: Print your name, and student ID number in the space provided. You may use Armstrong and any notes taken from class, but not additional texts and resources. Read each question carefully. Try to be as precise as possible when writing your proofs. Correct answers without justification will receive little or no credit. There are 6 questions. There is no time limit.

On this take home exam, there are two parts. The first is similar to problem sets done for homework. It is worth 70 of 100 points. The second part is a guided set of problems through a new topic. It is worth 30 out of 100 points. Several of the questions are quite difficult, mainly toward the end, so you are not expected to finish all parts of all questions.

Question	Score	Maximum
1		10
2		10
3		10
4		10
5		10
6		10
Total		60

1. Determine if the following sets G with the indicated operation form a group by checking the group axioms. If not, point out which of the group axioms fail.

(a) $G =$ set of all integers \mathbb{Z} , with $a * b = a - b$.

(b) $G =$ set of all integers \mathbb{Z} , with $a * b = a + b + ab$.

(c) $G =$ set of all rational numbers $\mathbb{Q} \neq -1$, with $a * b = a + b + ab$.

(d) $G =$ set of all integers modulo 47, $\mathbb{Z}/47\mathbb{Z}$, with $a * b = ab \pmod{47}$.

2. Prove that every cyclic group G is abelian.

3. (a) Express the following elements of S_8 , the symmetric group of order $8!$, as a product of transpositions.

i. $(1\ 4\ 3)$

ii. $(2\ 8)(6\ 5\ 4)(4\ 5)$

- (b) Express the following elements of S_{11} , the symmetric group of order $11!$, as a product of disjoint cycles.

i. $(7\ 4\ 3\ 8)(6\ 5\ 3)(2\ 1)(1\ 4\ 2)$

ii. $(6\ 5)(5\ 7)(8\ 4)(3\ 1\ 2)(3\ 5\ 11)$

4. Suppose that for a group G ,

$$(ab)^3 = a^3b^3 \quad \text{and} \quad (ab)^5 = a^5b^5 \quad \text{for any } a, b \in G.$$

Prove that G is abelian.

5. (a) Prove that for any finite group G , given any element $a \in G$, there is a positive integer n depending on a such that $a^n = e$.

- (b) Use the result above to show that for a finite group G , there is a positive integer m such that $a^m = e$ for all $a \in G$.

6. Find a shuffle of a deck of 52 cards that requires 42 repeats to return the cards to their original order.