Math 109, Winter 2005
Proof Workshop Problems
Thursday, January 6

1. Prove the following statements about sums of integers:
   (a) $1 + 2 + 3 + \cdots + k = \frac{1}{2}k(k+1)$
   (b) $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(n+2)$
   (c) Can you find a similar expression for the sum of the first n cubes?
2. Prove that if p and q are odd integers, then $p \cdot q$ is odd.
3. Prove that if p and q are integers such that $p \cdot q$ is even, then either p is even or q is even.
4. Prove that $\sqrt{2}$ is irrational. Can you generalize your proof for other integers $n \neq 2$? Why or why not?
5. Given a group G, prove that $(a^{-1})^{-1} = a$ for all elements $a \in G$.
6. If G is any group and $a, b, c \in G$ show that if $a * b = a * c$, then $b = c$ and if $b * a = c * a$ then $b = c$.
7. If G is a finite group, show that for each $a \in G$ there is an integer n (depending on a) such that $a^n = e$, the identity in the group.
8. Show that any group of size 4 or less is abelian.
9. Prove that $n^3 - n$ is always divisible by 3.
10. How many ways are there of choosing 2 hats from among 38 hats? What about from among n hats?
11. Take a 4×4 grid of squares and remove the bottom left and upper right squares. Can you tile the resulting shape with 2×1 squares (often referred to as dominoes)? What about 5×5? What about 8×8?