More Midterm II Review Problems
Math 109, Winter 04-05

1. Show that a homomorphism $\varphi : G \to G'$ is one-one if and only if $ker(\varphi) = \{e\}$
2. Prove that if $Z(G)$ is the center of G, then $Z(G) \lhd G$.
3. Let $G = \mathbb{R}^\times$ and $N = \{-1, 1\}$. Show that G/N is isomorphic to $(\mathbb{R}^+)$ the group of positive real numbers under multiplication.
4. If G is abelian, and φ is a homomorphism ONTO G', prove that G' is abelian.
5. If G is an abelian group and N is a subgroup of G, show that G/N is abelian.
6. If φ is a homomorphism of G ONTO G', and $N \lhd G$, show that $\varphi(N) \lhd G'$.
7. Let G be a group such that all subgroups are normal in G. If $a, b \in G$, prove that $ba = a^j b$ for some j.
8. An automorphism is an isomorphism of G with itself. One such example is conjugation by a fixed element $g \in G$. If ϕ is an automorphism of G and $N \lhd G$, prove that $\phi(N) \lhd G$.
9. Prove that the set of all automorphisms of G forms a group under composition.
10. If G is an abelian group of order $p_1 p_2 \cdots p_k$, a product of distinct primes, prove that G is cyclic.
11. A subgroup T of G is called characteristic if $\phi(T) \subseteq T$ for all automorphisms ϕ of G. Prove that if M is characteristic in G, then $M \lhd G$.
12. Show that $Z(G)$ is a characteristic subgroup of G.
13. If G is a finite group, $H \leq G$ such that $|G|$ does not divide $(|G|/|H|)!$. Prove that H contains a proper normal subgroup of G.
14. Use the previous problem to show that there are no simple groups of order 21 (Hint: use Cauchy's theorem).
15. Let G be a finite group and ϕ an automorphism of G such that $\phi(x) = x^{-1}$ for more that 3/4 of the elements of G. Prove that $\phi(y) = y^{-1}$ for al y in G, so G is abelian.