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A fundamental question in Riemannian geometry is to find canonical metrics on a given smooth manifold. In the 1980s, R. Hamilton proposed an approach to this question based on parabolic partial differential equations. The goal is to start from a given initial metric and deform it to a canonical metric by means of an evolution equation. There are various natural evolution equations for Riemannian metrics, including the Ricci flow and the conformal Yamabe flow. In this survey, we discuss the global behavior of the solutions to these equations. In particular, we describe how these techniques can be used to prove the Differentiable Sphere Theorem.

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