Manifolds with 1/4-pinched curvature are space forms, Journal of the American Mathematical Society 22, 287–307 (2009) (joint with R. Schoen)

In 1926, Hopf proved that any compact, simply connected Riemannian manifold with constant curvature 1 is isometric to the standard sphere. Motivated by this result, Hopf posed the question whether a compact, simply connected manifold with suitably pinched curvature is topologically a sphere. This question has been studied by many authors over the past six decades, a milestone being the Topological Sphere Theorem proved by M. Berger and W. Klingenberg in 1960.

In this paper, we give a proof of the Differentiable Sphere Theorem. This theorem classifies all manifolds with 1/4-pinched curvature up to diffeomorphism. The distinction between homeomorphism and diffeomorphism is significant in light of the exotic spheres constructed by J. Milnor.

The proof uses the Ricci flow technique introduced by R. Hamilton. A key ingredient in the proof is the fact, also established in this paper, that nonnegative isotropic curvature is preserved by the Ricci flow in all dimensions. This latter condition was first studied by M. Micallef and J.D. Moore in the context of minimal surface theory.