

**MIDTERM 2 BONUS PROBLEMS**

*Instructions:* Complete the following problems. Each is worth the stated number of points to be added to your final Midterm 2 grade. There is no partial credit awarded for these problems: either your solution is entirely correct (in which case you will get full credit) or it is not entirely correct (in which case you will get zero credit). Make sure your solutions are completely justified and neat: an answer without justification is counted as incorrect! Some problems may ask you to prove a result; if you are unsure of how one proves a result, just come talk to me. *You should not use any resources outside the text and your notes (and me).*

- (1) The following problems deal with linear spaces, an abstraction of the vector spaces we've talked about so far. To familiarize yourself with these spaces and the properties they have, read Chapter 4.1 of Bretscher.
  - (a) ( $\frac{1}{2}$  pt) Complete problem 4.1.47 from Bretscher's *Linear Algebra with Applications*.
  - (b) (1 pt) Let  $\mathcal{O}$  be the space of all odd functions from  $\mathbb{R}$  to  $\mathbb{R}$  and  $\mathcal{E}$  the space of all even functions from  $\mathbb{R}$  to  $\mathbb{R}$ . You showed these sets were subspaces of  $F(\mathbb{R}, \mathbb{R})$  in the previous problem. Show that  $F(\mathbb{R}, \mathbb{R}) = \mathcal{O} + \mathcal{E}$  in the sense that any function  $f : \mathbb{R} \rightarrow \mathbb{R}$  can be written as a sum of an even function and an odd function.
  - (c) ( $\frac{1}{2}$  pt) Show that  $\mathcal{O} \cap \mathcal{E} = \{0\}$ , where here 0 represents the 0 function (for any input, the output of the zero function is 0). Together with the previous problem, this means that  $F(\mathbb{R}, \mathbb{R}) = \mathcal{E} \oplus \mathcal{O}$ .
  - (d) (1 pt) Complete problem 5.1.18 from Bretscher's *Linear Algebra with Applications*.
- (2) (2 pts) Complete problem 5.3.64 from Bretscher's *Linear Algebra with Applications*.
- (3) (1 pt) Complete problem 5.4.39 from Bretscher's *Linear Algebra with Applications*.
- (4) (1 pt) Complete problem 6.1.59 from Bretscher's *Linear Algebra with Applications*. (You'll need to know mathematical induction for this. If you haven't come across this before, you can still do the problem! Just talk to me about induction.)
- (5) (1 pt) Complete problem 6.2.54 from Bretscher's *Linear Algebra with Applications*.
- (6) (1 pt) Complete problem 7.3.48 from Bretscher's *Linear Algebra with Applications*.