MATH 83N: Proofs and Modern Mathematics
SYLLABUS

Official Course Description: How do mathematicians think? Why are the mathematical facts learned in school true? In this course students will explore higher-level mathematical thinking and will gain familiarity with a crucial aspect of mathematics: achieving certainty via mathematical proofs, a creative activity of figuring out what should be true and why. Through this seminar, students will prove some mathematical statements with which they may already be familiar; introduce new concepts and ways of thinking that illuminate known facts and help us to explore further; and learn how to carry out careful logical arguments and write proofs. The final project will be writing up, in stages, a logically more complicated proof or sequence of proofs.

This course is ideal for students who would like to learn about the reasoning underlying mathematical results, but hope to do so at a more manageable pace and level of abstraction than Math 61CM/DM offers, as a consequence benefiting from additional opportunity to explore the reasoning. Familiarity with one-variable calculus is useful since a significant part of the seminar develops it systematically from a small list of axioms. We also address linear algebra from the viewpoint of a mathematician, illuminating algebraic notions such as groups, rings, and fields. This seminar may be paired with Math 51 although that course is not a pre- or co-requisite.

Satisfies Ways requirement for Formal Reasoning (FR).

Teaching Staff:

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Instructor: Dr. Andras Vasy
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Lecture: MW 1:30-2:50pm | 380-381T

Textbooks: The primary texts for this class are

Affordability of Course Materials: Stanford University and its instructors are committed to ensuring that all courses are financially accessible to all students. If you are an undergraduate who needs assistance with the cost of course textbooks, supplies, materials and/or fees, you are welcome to approach us directly. If would prefer not to approach us directly, please note that you can ask the Diversity & First-Gen Office for assistance by completing their questionnaire on course textbooks & supplies: tinyurl.com/jpqbarn or by contacting Joseph Brown, the Associate Director of the Diversity and First-Gen Office (jlbrown@stanford.edu; Old Union Room 207). Dr. Brown is available to connect you with resources and support while ensuring your privacy.

- *Linear Algebra Done Right* by Sheldon Axler (3rd edition, ISBN: 0387982582)
- *Understanding Analysis* by Stephen Abbott (2nd edition, ISBN: 1493927116)

You should read them! (Relevant chapters are listed on the schedule.) This does not mean that they are "easy" to read. Math books are quite demanding on the reader, owing to the intrinsic difficulty of the material, so do not be surprised if you have to go slowly.

Both the Axler and Abbott texts are **freely available** to you as PDFs through Stanford's license with Springer International Publishing. We recommend that you print out the relevant pages, since we find that students more deeply engage with printed rather than electronic materials. (The FedEx store on campus will print and bind the pages for ∼$15.)

Course website: Course announcements, assignments, solutions, and grades will be posted on Canvas.

Grading Policy: Our goal is for you to develop your ability to understand and write mathematical proofs and to learn some basic mathematical structures. We expect everyone will do well in the class. The assignment categories are weighted as follows:

- Homework: 50% (lowest score dropped)
- Linear Algebra Project: 15%
- Real Analysis Project: 20%
- Daily Quizzes: 10%
- Presentation: 5%

Homework & Presentation: The only way to learn mathematics is to do mathematics! The homework problems are an integral part of the course; they are the best and most reliable way to check your progress. Problems will range from fairly standard computations to routine applications of the definitions and formulae, to more difficult problems which will require more thought.

You'll have a problem set every week, due in class on Wednesdays. Please work out problems neatly—don't hand in your scratch work. Homework will be graded for clarity of exposition as well as correctness. Additionally, your homework must be stapled (no paperclips, please!) without the rough edges from tearing out of a spiral notebook.

One course goal is to sharpen your mathematical writing skills, and homework is a place to practice. Every week on or two problems will be marked for you to focus on your mathematical exposition. If you'd like to, you could type up these problems using LaTeX, a typesetting system that is standard for scientific documents; however, you don't need to. On these problems, you will receive feedback accordingly.

Additionally, you or a classmate will give a short presentation on each of these marked problems in class on Wednesdays. The date of your presentation will be assigned within the first two weeks.

We encourage you to form study groups and work together. A good strategy is to try each problem yourself first, then get together with others to discuss your solutions and questions, and finally *write up the solutions yourself.*

The lowest score will be dropped to accommodate exceptional situations such as a serious illness. Because the lowest score is dropped, you can miss one assignment without penalty. No late homework will be accepted, and no make-up homework will be given.

Writing Projects: There will be two writing projects in this course. In these writing projects you will read about some topic that we didn't cover in this course, and write a short expository paper. Essentially, you should think of it as writing a small section of a textbook.

For the first project, you'll write a short paper ($\sim$ 3 pages) on some topic from linear algebra. For the second project, you'll write a paper ($\sim$ 6 pages) on some topic from real analysis.

You'll have to type both of these assignments. We recommend that you use LaTeX, a typesetting system that is standard for scientific documents. We will provide sample LaTeXfiles for you to edit. However, there's a learning curve on LaTeX, and you are welcome to use a standard word processor with a mathematical plug-in if you'd prefer. More details will be given soon!

Schedule: This course is structured with the expectation that you will attend every lecture. Of course, sometimes an absence is necessary. In such a situation, you should contact a classmate to get notes and other information for the class you missed.

We will have 20 lectures in total. Here is a tentative schedule, which may be adjusted as the quarter goes on.

- Week 1: Groups, rings, fields (Notes)
- Week 2: Vector spaces (Axler Chapter 1.B-C (15 pages))
- Week 3: Vector spaces (Axler Chapter 2 (24 pages))
- Week 4: Linear maps (Axler Chapter 3.A-B (18 pages))
- Week 5: Linear maps (Axler Chapter 3.C-D (21 pages))
- Week 6: Real numbers (Abbott Chapter 1 (38 pages))
- Week 7: Sequences and series (Abbott Chapter 2 (46 pages))
- Week 8: Basic topology of the reals (Abbott Chapter 3 (26 pages))
- Week 9: Continuity (Abbott Chapter 4 (34 pages))
- Week 10: Differentiation (Abbott Chapter 5 (24 pages))

Typical Week: This class will be taught in a seminar style rather than lecture style. That means that is much more participatory and engaging!

Participation begins with preparation. You preparation for the week begins on your own with weekly readings. This is an opportunity to practice your skills of reading math textbooks. You don't have to understand everything!

At the beginning of every class there will be a short quiz (<5 minutes) in which you'll be asked to pull out a piece of paper and write down a particular definition or statement of a theorem from the previous lecture. Learning mathematics is like learning a language; knowing definitions and statements of theorems are essential for being fluent.

On Wednesdays, immediately after the quiz, 1-2 of you will each give a short ($\sim$ 5 minute) presentation on some assigned homework problem.

The rest of the class is a combination of lecture and time to work in groups of 3-4 on problems. Particularly thorny problems may be discussed in the last 10 minutes of class.

Supplemental Textbooks and other Resources: In addition to the required textbooks, the following textbook gives a systematic approach for teaching students how to read, understand, think about, and do proofs. We recommend it! The approach is to categorize, identify, and explain the various techniques that are used repeatedly in all proofs. It is available on 4-hour reserve in the library. The sections on mathematical quantifiers ($\forall$, $\exists$) may be relevant for our section on real analysis.

- *How to Read and Do Proofs: An Introduction to Mathematical Thought Processes* by Daniel Solow (6th edition, ISBN: 1118164024)

You are encouraged to attend the office hours provided by the instructors and course assistant.

Another resource which may be of use is Counseling and Psychological Services. See vaden.stanford.edu/caps-and-wellness.

Students with Documented Disabilities: Students who may need an academic accommodation based on the impact of a disability must initiate the request with the Office of Accessible Education (OAE). Professional staff will evaluate the request with required documentation, recommend reasonable accommodations, and prepare an Accommodation Letter for faculty dated in the current quarter in which the request is made. Students should contact the OAE *by the end of the first week of the quarter*, since timely notice is needed to coordinate accommodations. The OAE is located at 563 Salvatierra Walk (723-1066, studentaffairs.stanford.edu).

Computers: If you wish to use a computer in class, you must speak with each of us first.

Academic Integrity: The Honor Code articulates Stanford University's expectations of students and faculty in establishing and maintaining the highest standards in academic work. Examples of conduct that have been regarded as being in violation of the Honor Code (and are most relevant for this course) include copying from another's examination paper or allowing another to copy from one's own paper; plagiarism; revising and resubmitting an exam for regrading, without the instructor's knowledge and consent; representing as one's own work the work of another; and giving or receiving aid on an academic assignment under circumstances in which a reasonable person should have known that such aid was not permitted. See communitystandards.stanford.edu for more information on the Honor Code.

Important Dates:

First Day of Classes	September 24
Add/Drop Deadline	October 12
Course Withdrawal & Change of Grading Basis Deadlines	November 16
Thanksgiving Recess	November 19-23
Last Day of Classes, Last Day to Arrange an Incomplete	December 7