A Causal Five-Field Theory of Dissipative Relativistic Fluid Dynamics

Heinrich Freistühler (University of Konstanz)

In the absence of dissipation, relativistic fluid dynamics is governed by the Euler equations

$$\frac{\partial}{\partial x^\beta} T^{\alpha\beta} = 0, \quad \frac{\partial}{\partial x^\beta} N^\beta = 0,$$

which are a five-field theory: Its constituents, the energy-momentum tensor and the particle number density current,

$$T^{\alpha\beta} = (\rho + p)U^\alpha U^\beta + pg^{\alpha\beta} = 0, \qquad N^\beta = nU^\beta,$$

are given in terms of the fluid's velocity U^α, energy density ρ, pressure p, and particle number density n. As the velocity satisfies $U^\alpha U_\alpha = -1$ and ρ, p, n are constrained by an equation of state, the state description (U^α, ρ, p, n) has five degrees of freedom, and the Euler equations are five partial differential equations which determine the spatiotemporal evolution of these five fields from general initial data.

Regarding the modeling of dissipation, i.e., viscosity and heat conduction, various theories have been suggested over the last almost eight decades. This talk deals with the question of whether dissipative relativistic fluid dynamics can be properly modeled by a *causal* five-field theory

$$\frac{\partial}{\partial x^\beta} (T^{\alpha\beta} + \Delta T^{\alpha\beta}) = 0, \qquad \frac{\partial}{\partial x^\beta}(N^\beta + \Delta N^\beta) = 0,$$

with dissipation tensors $\Delta T^{\alpha\beta}, \Delta N^\beta$ that are linear in the gradients of the five fields ("relativistic Navier-Stokes"). The question is answered in the affirmative. The proposed formulation is intimately related to the theory of second-order symmetric hyperbolic systems developed by Hughes, Kato, and Marsden on the one hand and to the classical (non-causal) descriptions given by Eckart and Landau on the other.

(Joint work with B. Temple, UC Davis)