Bay Area Microlocal Analysis Seminar

Wednesday, November 17th, at Stanford

1:15-2:15pm, Room 380D

Blow-up solutions on a sphere for the 3d quintic NLS in the energy space

JUSTIN HOLMER
Brown University

Abstract
Solutions to the focusing nonlinear Schroedinger (NLS) equation $i\partial_t u + \Delta u + |u|^{p-1}u = 0$ for nonlinearities between mass-critical ($p = 1 + 4/d$) and energy-critical ($p = (d+2)/(d-2)$) can blow-up in finite time. In the mass-critical setting, the blow-up occurs on a discrete (dimension zero) set whereas in the mass-supercritical ($p > 1 + 4/d$) setting, the blow-up can occur on sets of positive dimension. Using microlocal methods, we first prove that the log-log blow-up solutions studied by Merle-Raphael (2001-2005) with single blow-up point to the mass-critical equation remain regular in the energy space away from the blow-up point, resolving a conjecture of Raphael-Szeftel (2008). We are thus able to insert such solutions into higher-dimensional equations under symmetry assumptions; such equations will be mass-supercritical. In particular, we construct a large class of radial solutions that blow-up on a sphere for the three-dimensional energy-critical NLS. This is joint work with Svetlana Roudenko. We also discuss some other recent work in the field.

AND

3-4pm, Room 383N

The geodesic X-ray transform in presence of caustics

PLAMEN STEFANOV
Purdue University

Abstract
We study geodesic type of X-ray transforms $X$ locally, near a geodesic segment with conjugate points. In the case of the sphere, we can have exact cancellations that $X$ cannot recover. We study the more common case of fold type of singularities of the exponential map. We analyze the microlocal invertibility (or not) of $X$. We show that cancelations of singularities always happens in 2D, at least of a finite order. In 3D, we give examples of cancellations and examples where we can invert $X$ microlocally. We analyze $X^*X$ and show that it is a sum of a pseudodifferential operator of order $-1$ and an FIO of order $-n/2$ with a Lagrangian given by the conormal bundle of the conjugate locus. The latter may or may not be of graph type.

http://math.stanford.edu/~andras/PDE/PDE.html

Organizers: Daniel Tataru, Andras Vasy & Maciej Zworski