Bay Area Microlocal Analysis Seminar

Wednesday, November 17th, at Stanford

1:15-2:15pm, Room 380D

Blow-up solutions on a sphere for the 3d quintic NLS in the energy space

JUSTIN HOLMER
Brown University

Abstract

Solutions to the focusing nonlinear Schroedinger (NLS) equation $i\partial_t u + \Delta u + |u|^{p-1}u = 0$ for nonlinearities between mass-critical ($p = 1 + 4/d$) and energy-critical ($p = (d+2)/(d-2)$) can blow-up in finite time. In the mass-critical setting, the blow-up occurs on a discrete (dimension zero) set whereas in the mass-supercritical ($p > 1 + 4/d$) setting, the blow-up can occur on sets of positive dimension. Using microlocal methods, we first prove that the log-log blow-up solutions studied by Merle-Raphael (2001-2005) with single blow-up point to the mass-critical equation remain regular in the energy space away from the blow-up point, resolving a conjecture of Raphael-Szeftel (2008). We are thus able to insert such solutions into higher-dimensional equations under symmetry assumptions; such equations will be mass-supercritical. In particular, we construct a large class of radial solutions that blow-up on a sphere for the three-dimensional energy-critical NLS. This is joint work with Svetlana Roudenko. We also discuss some other recent work in the field.

AND

3-4pm, Room 383N

The geodesic X-ray transform in presence of caustics

PLAMEN STEFANOV
Purdue University

Abstract

We study geodesic type of X-ray transforms X locally, near a geodesic segment with conjugate points. In the case of the sphere, we can have exact cancellations that X cannot recover. We study the more common case of fold type of singularities of the exponential map. We analyze the microlocal invertibility (or not) of X. We show that cancelations of singularities always happens in 2D, at least of a finite order. In 3D, we give examples of cancellations and examples where we can invert X microlocally. We analyze X^*X and show that it is a sum of a pseudodifferential operator of order -1 and an FIO of order $-n/2$ with a Lagrangian given by the conormal bundle of the conjugate locus. The latter may or may not be of graph type.

`http://math.stanford.edu/~andras/PDE/PDE.html`

Organizers: Daniel Tataru, Andras Vasy & Maciej Zworski