MATH 220: PROBLEM SET 1
DUE THURSDAY, OCTOBER 1, 2009

Problem 1. Classify the following PDEs by degree of non-linearity (linear, semi-linear, quasilinear, fully nonlinear):

(1) $(\cos x)u_x + u_y = u^2$.
(2) $uu_{tt} = u_{xx}$.
(3) $u_x - e^x u_y = \cos x$.
(4) $u_{tt} - u_{xx} + e^u u_x = 0$.

Problem 2.

(1) Solve
$$u_x + (\sin x)u_y = y,\ \ u(0, y) = 0.$$
(2) Sketch the projected characteristic curves for this PDE.

Problem 3.

(1) Solve
$$yu_x + xu_y = 0,\ \ u(0, y) = e^{-y^2}.$$
(2) In which region is u uniquely determined?

Problem 4.

(1) Solve $u_x + u_t = u^2,\ \ u(x, 0) = e^{-x^2}$.
(2) Show that there is $T > 0$ such that u blows up at time T, i.e. u is continuously differentiable for $t \in [0, T)$, x arbitrary, but for some x_0, $|u(x_0, t)| \to \infty$ as $t \to T-$. What is T?

Problem 5. Solve
$$u_t + uu_x = 0,\ \ u(x, 0) = -x^2$$
for $|t|$ small.

Problem 6. Consider the PDE
$$u_t + uu_x = 0,\ \ u(x, 0) = \phi(x).$$
Suppose that $\phi' \geq -C$, where $C > 0$. Show that the PDE has a C^1 solution on $\mathbb{R}_x \times [0, \frac{1}{C})_t$. Show also that for $t \in [0, \frac{1}{C})$, u_x satisfies the estimate
$$u_x(x, t) \geq \frac{1}{t - C^{-1}}.$$
(Note that the right hand side is negative!) (Hint: Consider the difference quotients $\frac{u(\xi_2(t),t)-u(\xi_1(t),t)}{\xi_2(t)-\xi_1(t)}$, where $x = \xi_j(t)$ are the projected characteristic curves emanating from the point x_j on the x axis.)