Do the following problems from the textbook: Lesson 17: 1,3,4, Lesson 18.1,3, Lesson 19.1,2, as well as the following problem:

**Problem 1.** Calculate the Fourier transform of the function \( f(x) = e^{-ax^2}, a > 0, \) as follows. First rewrite the integral

\[
\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-i\xi x} e^{-ax^2} \, dx = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-(ax^2 + i\xi x)} \, dx
\]

by completing the square in \( x \). Pull out a Gaussian factor in \( \xi \), i.e. \( e^{-b\xi^2} \) for some \( b > 0 \), so that you are left with an integral of the form

\[
\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-a(x+i\alpha \xi)^2} \, dx
\]

for some \( \alpha \). Then show that this is independent of \( \xi \) by showing that its \( \xi \) derivative is 0: when you differentiate, do it under the integral sign, and rewrite to have an expression (still in the integrand) that is of the form \( \frac{\partial}{\partial \xi} \) for some \( g \) which decays as \( |x| \to \infty \), so by the fundamental theorem of calculus the integral (in \( x \)!) is 0. Finally, evaluate

\[
\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-a(x+i\alpha \xi)^2} \, dx
\]

when \( \xi = 0 \): there is a standard trick in calculus for doing this. (Take the square of this expression, writing the two factors as integrals in different variables \( x \) and \( y \), and convert to polar coordinates.)