(1) Compute the character table of A_4.

(2) Let $X = (12) - e \in \mathbf{C}S_3$ and $Y = (123) + 3(12) \in \mathbf{C}S_3$. Compute the product XY. Now compute the images of X, Y, XY under the isomorphism $\mathbf{C}S_3 \to M_2(\mathbf{C}) \times \mathbf{C} \times \mathbf{C}$ that we discussed in class.

(3) Let N be the group of upper triangular matrices in $\mathrm{GL}_3(\mathbf{F}_p)$ with all diagonal entries equal to 1. Let Z be the center of N; it is a cyclic p-group.
   (a) Describe the conjugacy classes in N.
   (b) Show that N has exactly p^2 one-dimensional representations, and describe them explicitly.
   (c) Prove that there are $p - 1$ remaining representations of N, that they are all p-dimensional, and for each such representation V_i the trace χ_i is *zero* on any elements $\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$ unless $a = c = 0$.

(4) Let $G \subset \mathrm{GL}_n(\mathbf{C})$ be a finite group, so that G acts on $V = \mathbf{C}^n$. Prove that every irreducible representation of G occurs in $V^{\otimes k}$ for some $k \geq 1$.

(5) Let $X = e + (12) + (123) + (1234) \in \mathbf{C}S_4$. Prove that there are only finitely many $\lambda \in \mathbf{C}$ such that $X + \lambda$ is noninvertible. Find all such λ.

(6) (This one is tricky:) Let P be the representation of S_n on $\{(x_1, ..., x_n) \in \mathbf{C}^n : \sum x_i = 0.\}$ Prove that $P \otimes P$ s the sum of exactly four irreducible non-isomorphic representations.