Conference on Algebraic Topological Methods in Computer Science

July 30 - August 3, 2001 at Stanford University

Over the past 5 to 10 years, there has been an increasing interest in the potential applications of algebraic topology, particularly in the area of computing. This new interest derives from several directions.

- The recognition that within the area of computational geometry, the methods of algebraic topology can provide qualitative and shape information which isn't available from other methods.
- Algebraic topology provides a tool for visualization and feature identification in high dimensional data.
- Algebraic topology is an extremely useful framework for analyzing problems in distributed computing, such as concurrency.
- Algebraic topology has been demonstrated to be a useful tool in solving combinatorial problems of relevance to computing.

The goal of the conference is to bring together people working in different areas of applications of algebraic topology, but who have in common an interest in algebraic topology. It is our feeling that this is an extremely fertile area at the moment, and that this meeting will be intellectually extremely fruitful.

Organizers: Gunnar Carlsson (Stanford University), Rick Jardine (University of Western Ontario)

Invited speakers:

John Baez	(Math, UC Riverside)	Marco Grandis	(Dip. di Mat., Genova)
Marshall Bern	(Xerox PARC)	John Harer	(Math Dept, Duke University)
Anders Björner	(Royal Inst. of Tech., Stockholm)	Joel Hass	(Math Dept, UC Davis)
Tamal Dey	(CS, Ohio State)	Maurice Herlihy	(CS, Brown University)
Herbert Edelsbrunner	(CS, Duke University)	Reinhard Laubenbacher	(NMSU)
David Eppstein	(CS, UC Irvine)	Laszlo Lovasz	(Microsoft)
Michael Freedman	(Microsoft)	Vaughan Pratt	(CS, Stanford)
Philippe Gaucher	(CNRS, Strasbourg)	Christian Reidys	(Los Alamos National Lab)
Eric Goubault	(CEA, France)	Bernd Sturmfels	(Math Dept, UC Berkeley)
Jean Goubault-Larrecq	(ENS Cachan)	Noson Yanofsky	(CS, Brooklyn College)

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ATMCS Conference Program

	July 30	July 31	August 1	August 2	August 3
9:30-10:30	Baez	Sturmfels	Reidys	Herlihy	Goubault
11:00-12:00	Pratt	Lovasz	Edelsbrunner	Eppstein	Yanofsky
1:15-2:50	Cont. Talks	Cont. Talks	Cont. Talks	Prob. Sess.	Cont. Talks
3:00-4:00	Hass		Björner	Bern	Gaucher
4:15-5:15	Dey	Harer	Grandis	Laubenbacher	Goubault-Larrecq

John Baez - UC Riverside
"Categorification and Computation"

Categorification is the process of replacing equations by isomorphisms. This lets us keep track of the "path of computation" going from x to y when we prove that x = y. Iterating the process of categorification leads naturally to the concept of an "n-category": a structure with objects, morphisms between objects, 2-morphisms between morphisms and so on to the nth level. After a brief review of recent progress in n-category theory, we describe how the bar construction from algebraic topology can be used to automatically categorify a large class of mathematical structures - starting with the natural numbers and working on up.

Vaughan Pratt - Stanford University
"Algebraic topology via point set topology"

Whereas algebraic topology deals with the hierarchical combinatorics of n-cells connected via their faces, point set topology is based on the seemingly unrelated notions of neighborhood and limit. The absence of visible algebra from the latter notwithstanding, it is possible to base the former entirely on the latter simply by relaxing two conditions. We have recently applied this idea to refine our 1991 topological model of concurrent behavior in which n-cells serve as states. The talk will address aspects of this work.

Joel Hass - UC, Davis
"Geometry, Topology and Computational Complexity"

There are unexpected connections between the complexity of algorithms to recognize knots, the area of a smallest spanning disk for a curve, the number of triangles required to triangulate spanning surfaces, and the minimal genus of a spanning surface for a knot. This talk will explore some of these connections.

Tamal K. Dey - Ohio State University
"Topological Dimension of Shapes from Discrete Samples"

Applications in data visualization, learning theory, neural networks require to determine the topological dimension of a shape from a set of discrete samples. We argue that the topological dimension can be determined from the geometric structure of the Voronoi cells of the sample points under a suitable assumption on sampling density. Based on this analysis, a simple algorithm is devised which assigns a dimension k to a sample point if it belongs to a manifold of dimension k. The algorithm can be used in conjunction with a generalized reconstruction method to compute a piecewise linear approximation of the sampled shape. This research brings us to some topological questions that I shall present along with our empirical results.

Bernd Sturmfels - UC Berkeley
"Cell complexes in computational algebraic geometry"

Algorithmic problems involving polynomials can be reduced to the study of monomials using the method of Gröbner bases. In this talk we illustrate the use of cell complexes labeled by monomials in the design of efficient algorithms for computing invariants (e.g. dimension) of zero sets of polynomial equations. A special role is played by a simplicial complex first introduced by Herbert Scarf in the 1960's in the context of mathematical economics.

Laszlo Lovasz - Microsoft
"Topological nonexistence results in complexity theory and combinatorics"

Proving the nonexistence of efficient algorithms in different computational models is among the most basic and most difficult problems in complexity theory. One needs to define a measure of "complexity" of the problem, show that this is high on appropriate instances, and also show that an efficient algorithm would imply an upper bound that is too good.

If the complexity measure is very simple (e.g., number of edges in a graph), then it leads to poor bounds. If it is too complicated (e.g. the length of the best algorithm solving the problem), then it is impossible to estimate. Algebraic topology (like connectivity or Betti numbers of appropriate simplicial complexes associated with the problem instance), provides an interesting middle ground, which leads to non-trivial results. Similar methods can be applied to proving the nonexistence of combinatorial objects, like colorings of graphs.

John Harer - Duke University
"Hierarchical Morse Complexes"
(Joint with Herbert Edelsbrunner and Afra Zomorodian)

We present algorithms for constructing a hierarchy of increasingly coarse Morse Complexes as a tool to analyze and visualize 2-d data sets. We also describe how to simplify these Morse complexes by cancelling pairs of critical points in order of increasing persistence and illustrate the utility of this approach in analyzing geographic terrain data.

Christian Reidys - Los Alamos National Lab
"Sequential Dynamical Systems:
A framework for theoretical foundations of computer simulations"

Computer Simulations exhibit a generic structure: they consist of a collection of entities that have states and corresponding ``dependency'' links among themselves and some schedule according to which their states are being updated. The straightforward formalization of this structure interprets an entity with a Boolean function whose variables are the states of all entities that are adjacent to the given one. The update of the system consists in the composition of these functions and its iteration yields the sequential dynamical system (SDS).
More precisely, an SDS consists of the following data:: (a) a finite (labeled) graph Y with vertex set 1,...,n where each vertex has a binary state, (b) a vertex labeled multi-set of functions F(i,Y) which map binary n-tuples into binary n-tuples and (c) a permutation, p. The function F(i,Y) updates the binary state of vertex i as a function of the states of vertex i and its Y-neighbors and leaves the states of all other vertices fixed. One main theme in SDS research is to deduce properties of the SDS based on the structure of the base graph, the functions involved and the underlying schedule. In this talk we will outline the research on SDS and summarize basic results. In particular we will give a cohomological interpretation of fixed points of SDS and study covering maps over n-cubes in the context of a reduction result in which, under certain conditions, a simpler SDS can be constructed which exhibits key features of the given one. Finally, we show how the SDS framework is used in concrete simulation projects.

Herbert Edelsbrunner - Duke University
"Alpha shapes, molecules, size, and connectivity"

Alpha shape are (underlying spaces of) complexes in a filtration of the Delaunay triangulation. Each alpha shape is the dual of a molecule represented as a space filling diagram on some scale level of perception. The volume and surface area of the molecule can be computed using short inclusion-exclusion formulas derived from the corresponding alpha shape. The connectivity of the molecule can be expressed by the homology groups of the corresponding alpha shape. We introduce persistent homology groups to capture the scale-dependence of a topological feature.

Anders Björner - Royal Inst. of Technology, Stockholm
"Topological lower bounds for decision trees"

One idea for obtaining lower bounds to the length of algorithms is to somehow interpret the computational process topologically and then rely on bounds provided by algebraic topology. In other words, we want to reduce computational complexity to topological complexity, which can then hopefully be estimated.
This idea has been applied with some success to algorithms that can be modeled by decision trees with algebraic tests (e.g. linear tests such as $x_i > x_j$?) at the inner nodes. In the talk I will survey and exemplify results of this kind, where the topological complexity measure is Euler characteristic or sum of Betti numbers.

Marco Grandis - Dip. di Mat., Genova
"Ordinary and directed combinatorial homotopy applied to image analysis and concurrency"

Combinatorial homotopical tools developed in previous works, and consisting essentially of intrinsic homotopy theories for simplicial complexes and *directed* simplicial complexes, can be applied to explore mathematical models epresenting images, or directed images, or concurrent processes.
An image, represented by a metric space X, can be explored at a variable resolution e > 0, by equipping it with a structure t_e(X) of simplicial complex depending on e; this complex can be further analysed by homotopy and homology groups. Loosely speaking, these objects detect - in degree n - singularities which can be captured by an n-dimensional grid, with edges bound by e; this works equally well for continuous or discrete regions of euclidean spaces.
Similarly, a directed image, represented by an oriented graph with a metric on its set of vertices, produces a family of directed simplicial complexes f_e(X) and can be explored by the fundamental n-categories of the latter. Applying the same oriented tools to mathematical models of concurrent automata can likely be of interest.

Maurice Herlihy - Brown University
"ALGEBRAIC TOPOLOGY AND DISTRIBUTED COMPUTATION"

In the past several years, a number of researchers have successfully applied techniques from Algebraic Topology to solve a number of long-standing open problems in the theory of distributed and concurrent computation. This talk will describe the some basic problems in distributed computing, and how to solve them using notions from elementary Algebraic Topology. We will describe some open problems and possible future directions.

David Eppstein - UC Irvine
"Topological problems in hexahedral meshing"

We consider the problem of subdividing a polyhedral domain in R^3 into cuboids meeting face-to-face. For topological subdivisions (cell complexes in which each cell is combinatorially equivalent to a cube, but may not be embedded as a polyhedron) and simply-connected domains, a simple necessary and sufficient condition for the existence of a hexahedral mesh is known: a domain with quadrilateral faces can be meshed if and only if there is an even number of faces. However, conditions for the existence of polyhedral meshes remain open, as do the topological versions of the problem for more complicated domain topologies.

Marshall Bern - Xerox PARC
"Regression Depth and Equivariant Topology"

Linear regression asks for an affine subspace (line, plane, etc.) that best fits a set of data points in R^d. I will describe a new approach to robust regression, suggested by Peter Rousseeuw and colleagues, and its connection to topology. I will also give a quick survey of the use of results from equivariant topology in computational geometry.

Reinhard Laubenbacher - NMSU
"A New Combinatorial Homotopy Theory for Graphs and Simplicial Complexes"

We present the construction of a bigraded family of groups associated to a graph, and a related construction for simplicial complexes. This theory resembles classical homotopy theory of spaces and satisfies many of the same properties. However, it depends heavily on the combinatorial structure of the objects; for instance, it is not invariant under subdivisions of simplicial complexes. We will also discuss applications of this theory in several contexts relevant to computer science.

Eric Goubault - CEA, France
"Algebraic Topology and Concurrency"

In this talk, I will present some of the "geometrical theories" used for modeling concurrent systems. All of these are based on a simple observation made by E. W. Dijkstra in 1968: in simple concurrent systems

such as (idealized) shared-memory machines, it is simple to associate a Euclidean space X and a (closed) partial order with a concurrent program, by attributing one coordinate to each process running in parallel, and by taking the partial order to be the usual component-wise ordering. This partial order reflects the way (global) time flows, respecting each local times (i.e. coordinate).
An essential feature of the model is that possible executions are continuous paths preserving the "flow of time". This means that executions, also called traces, are continuous maps from the unit segment to X which are monotonic (with the usual global ordering on the unit segment).
Another essential feature of the model is that if we do not have nice laws of evolution of the system, like in classical mechanics, we have good knowledge of what we call "forbidden regions" in X, i.e. points which cannot be states of the system. They are generated by synchronisations or mutual exclusions between processes, that want to access the same resources.
One simple observation is that the only thing that counts is homotopy classes of traces since deforming traces continuously does not affect the outcome of the computation. In fact this is not quite homotopy we need but a particular form called "directed homotopy" or "dihomotopy" which is a continuous deformation preserving the flow of time.
After a rather detailed part motivating the introduction of such a homotopy theory, I will present the programme of research as it is today, which is continuous collaboration with a number of other researchers, or based on work by others (V. Pratt, R. van Glabbeck, J. Gunawardena, M. Raussen, L. Fajstrup, S. Sokolowski, P. Gaucher, R. Brown etc.). Different formalisations (combinatorial, topological, categorical) of the problem will be presented, some of the main results of the theory will be shown together with their computer scientific applications.

Noson Yanofsky - Brooklyn College
"Algebraic Structure and Homotopy"

Algebraic theories are a categorical way of describing sets with extra algebraic structure (data types). A generalization to higher dimension leads to algebraic 2-theories which describe categories with extra structure. Within this higher dimensional universal algebra there is the flexibility to replace one algebraic structure with another (coherence theory). This flexibility is mirrored by the fact that one can place a Quillen model category structure on the category of algebraic 2-theories and 2-theory-morphisms. An exploration of the implied homotopy theory is used to better understand this universal algebra. We shall discuss minimal models of algebraic structure and their relationship with semantics of the 2-theory. We shall also talk of our work with similar ideas in other contexts such as (co-)monads and classical algebraic theories.

Philippe Gaucher - CNRS, Strasbourg
"Achronal simplexes and deformation of higher dimensional automata"

The formalization of higher dimensional automata (HDA) by strict globular omega-categories has led to a better understanding of the following problems : 1) understanding the deformations of HDA leaving invariant their computer-scientific properties like the presence or not of deadlocks ; 2) putting up an algebraic theory of these deformations. I will explain some of these ideas by illustrating with the omega-categorical case.

Jean Goubault-Larrecq - ENS Cachan
"Intuitionistic Modal Logic Proofs and Presheaves"

I will try to show some intimate connections between logic and geometry, which extends a famous connection discovered by Curry and Howard between proofs and programs to one between proofs, programs, and certain algebras of presheaves. The full array of connections reads:

> [Logic] [Computing Science] [Geometry]
> Formulae = Types = Presheaves
> Proofs = Programs = Presheaf morphisms
> Equivalence of proofs= Convertibility = Equality

I. Models
The cases of intuitionistic logic, and modal logics K, KT, K4 and S4 will be correspond to sets, graded sets, semi-simplicial sets, graded sets with degeneracies, and simplicial sets. (In fact, cubical sets, and

cubical sets with connections are also handled by the same logic S4.)
This extends through geometric realization functors to models of proofs in these logics, and therefore of corresponding programming paradigms, inside the realms of either Kelley spaces or preorders, including cpos. For example, the blackBox modality of S4 logic ("in every past") is both a suspension operator in computing, and the decalage functor in simplicial sets, and also some particular space of paths in topology. Its left adjoint whiteDiamond ("in some future") is Moggi's computational monad in programming, it builds spaces of cones in topology and is the lifting monad in cpos. This allows some form of topological, resp. order-theoretic, reasoning on programs up to convertibility.

II. Completeness Theorems

We will also show an equational completeness theorem in the case of the logics mentioned above, using so-called simplicial logical relations, and extending a theorem of Friedman's to the modal case. (This shows that equality of presheaf morphisms is indeed exactly equivalent to convertibility of programs, in the third line of the array above.) It is conjectured that this extends to a whole class of modal logics and presheaves built from rewrite systems on words. The latter construction displays intriguing connections with Squier's result on the existence of finitely presented monoids with a decidable word problem but not presented by any canonical rewrite system.

III. Kripke Semantics

If time permits, I will also discuss Kripke semantics for these logics, which are intuitionistic and therefore have a richer notion of Kripke models. For example, presheaf models of S4 proofs include morphisms from blackBox (F + G) to blackBox F + blackBox G, which is invalid in usual Kripke models for S4. I will develop modified Kripke models that allow for such pathological axioms. The question of exactly which Kripke models characterize the question of the existence of morphisms is still open.

Contributed Talks Schedule

	July 30	July 31	August 1	August 3
1:15-2:00	de Silva	Landi Kaczynski	Zomorodian Abrams	Hachimori
2:05-2:50	Pronk	Saalfeld Fajstrup	Raussen	Gonzale-Diaz

Vin de Silva - Stanford University
"Approximating finite sets by simplicial complexes"

In algebraic topology, the technique of viewing a topological space as a simplicial complex has a long and distinguished history; it is a way of representing continuous spaces (such as manifolds) as discrete objects. By contrast, in nonlinear data analysis one faces the problem of recovering the continuous structure of a space based on a discrete sample taken from it. Again, simplicial complexes are a natural candidate for bridging the gap. In this talk we discuss some of the mathematical issues which arise, leading to several open questions.

Dorette Pronk (joint work with R. Pare and R.J.M. Dawson)
"Adjoints and Undecidability"

Various familiar constructions can be interpreted as adding right (or left) adjoints to every arrow in a given category. For example, the category of relations and the category of spans are two categories that can be obtained from the category of sets by adding right adjoints subject to certain conditions. These constructions don't add the adjoint arrows freely to the category. Schanuel and Street decribe the result of adding freely adding adjoints to the category 2. We describe a construction Π_2 which adds right adjoints to all arrows in a category, which is free in the sense that every other construction that adds adjoints factors uniquely (up to 2- isomorphism) through this one.
The description of the 2-cells in the new 2-category $\Pi_2(C)$ involves a collection of representatives

with an equivalence relation, which is generated by a relation that is not transitive; so the question whether two given representatives are equivalent may not be decidable.
In this presentation we will show how for an arbitrary abacus A, one can construct a category $C(A)$ such that the equivalence relation on the 2-cells of $\Pi_2(C(A))$ simulates the transitions of the abacus, in such a way that determining whether two 2-cells are equivalent is the same as determining whether the abacus will halt, which shows that in general equality for such 2-cells is undecidable.
Besides these `toy'-categories taht we constructed specifically for the purpose of undecidability, we will also discuss the results for some more familiar categories such as sets and groups.

Claudia Landi - Universita di Reggio Emilia
"Morse homology for shape description"

The aim of this work is to show how the classical framework of Morse homology can be used to deal with the problem of shape recognition.
In computer vision, a possible approach to the problem of shape classification and recognition is that of representing shape properties of the given image by means of a shape descriptor.
Since objects appearance in images usually is not identical to a known prototype (due to position, orientation, noise and other distortions), shape descriptors are useful to detect those properties in an image which remain constant despite variability in appearance. Among shape descriptors, those of a geometric-topological nature seems to be very suitable when dealing with non-rigid objects. Various approaches have been proposed (e.g. by Siddiqi {\em et al.} (\cite{Siddiqi}), Kupeev and Wolfson (\cite{Kupeev}), Verri {\em et al.} (\cite{Verri}).
We propose a new topological approach to the problem of shape description (and in general suitable for the description of any data that can be modeled as a manifold), based on the classical theory of Morse homology.
The application of Morse theory to the analysis of data has already been proposed for example by Axen and Edelsbrunner \cite{Axen}, to deal with the problem of using audio to display the shape and connectivity of data set. This seems to indicate that Morse theory is an appropriate tool for dealing with topological questions in computer applications. Nevertheless our work differs from that of Axen and Edelsbrunner both in the setting of the problems (recognition rather than display of shape) and in the actual development of the methods.
Our method works as follows. Let us consider the set of all pairs (M,f), where M is a closed smooth manifold and $f:M\rightarrow \R$ is a Morse function. As it is well known, critical points of f give information about the global topology of M. Indeed M has the homotopy type of a CW-complex, with one cell of dimension λ for each critical point of f of index λ (see, e.g., \cite{Milnor}). Actually, it is possible to be more precise about how the cells corresponding to the various critical points of f glue together (see, e.g., \cite{Schwarz}). So, the critical points of f determine the homology of M. Therefore, it seems interesting to use classical Morse theory in the applicative task of describing shape of manifolds.
Of course, global topological information about M are a too rough descriptor of shape. In order to obtain a finer tool for shape description, we shall apply Morse theory to study the evolution of the homology at the different level sets $M^x=\{p\in M|f(p)\le x\}$. More precisely, to each such level set we shall associate a presentation of its homology given in terms of the critical points of f. Given a pair (M,f), the evolution of the presentations of the various $\tilde{H}_\lambda (M^x)$ portraits the development of the shape of the manifold, taking into account not only the topology of the various M^x, but also how the cells glue together. This is what we consider essential for studying the shape of (M,f).
Actually, it turns out that it is sufficient to consider only a finite number of level sets. Thus we have obtained a discrete description of the shape of M which is clearly more satisfactory than simply the global homology of M. Indeed, in this way we keep track of the possible presence of vanishing cycles. Moreover, by changing the function f with another Morse function, the homology of M does not change, while, in general, the homology of their level sets will be different. Therefore, if we see Morse functions f as shape properties of M, by varying f we obtain a collection of descriptors of M.
This approach can be considered as the development of a previous research (\cite{Cagliari}) on the categorical aspects of another shape descriptor of topological nature, called size function (see, e.g., \cite{Frosini}, \cite{Verri}). In such research, it has been brought out how size functions represent the evolution of the rank of the homology of degree 0 of the level sets of the manifold under study. Thus size functions appear not very indicated to describe multi-dimensional data. On the other hand, the usefulness of size functions have been shown in a number of tasks involving mono-dimensional data (e.g. recognition of monograms (\cite{FFLZ}), signatures (\cite{DFLo}), hand drawn sketches (\cite{CFFP}), hand-gestures (\cite{UV2})), motivating the present work.

Once we have obtained a description of (M,f) in terms of evolution of the presentations of the homology of M associated to f, we need the definition of some measure of the similarity between collections of presentations in order to be able to estimate similarity between two pairs (M,f) and (N,g). We show that this can be easily achieved by transforming each collection of presentations of Morse homology into a graph (more precisely a forest), called H_λ-forest, and by reducing the problem to that of comparing forests. The problem of matching graphs and trees is widely studied (see, e.g., \cite{Bunke}, \cite{Pelillo}).
We end the work by discussing the effective implementation of this method. An algorithm for the case of surfaces embedded in the Euclidean space is presented. Implementation of this algorithm can represent a step forward automatic recognition of two-dimensional data.

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Tomasz Kaczynski - Université de Sherbrooke, Canada
" Recursive Coboundary Formula for Cycles in Acyclic Chain Complexes"

Given an (m-1)-dimensional cycle z in a finitely generated acyclic chain complex (for example a triangulation of a polyhedron, cubical grid or a finite cellular complex) we want to construct an m-dimensional chain COB(z) whose algebraic boundary is z. The acyclicity of the chain complex implies that a solution exists (it is not unique!) but traditional linear algebra methods of finding it lead to a high complexity of computation. We are searching for more efficient algorithms based on geometric considerations.
The main motivation for studying this problem comes from the topological and computational dynamics, namely, from designing general algorithms computing the homomorphism induced in homology by a continuous map. This, for turn, is an essential step in computing such invariants of dynamical properties of a map as Conley index or Lefschetz number. Another potential motivation is in a relationship of our problem to the problem of finding minimal surfaces of closed curves.

Alan Saalfeld - Ohio State University
"Computational Algebraic Topology Opportunities in Computer Mapping"

Early in the development of computer mapping systems, scientists and mathematicians at the US Bureau of the Census, Harvard's Laboratory for Computer Graphics and Spatial Analysis, and elsewhere recognized and documented topological principles for cartography. The cell-like structure of map regions and their boundaries led naturally to an examination of the theory of finite cell topology for compact orientable surfaces with boundary (a paper map) or without boundary (a globe).

Cell topology theory, in turn, provided map-makers with many extremely useful results, including: (1) ways to test if a given combinatorial cell structure is even realizable as a surface of the desired type, (2) efficient ways to detect and correct any inconsistencies in the geometric location/feature positioning structure and the topological neighborhood structure, and (3) duality results that permit the same algorithmic structure for completeness-of-boundary tests and completeness-of-coboundary tests, tests that are needed to guarantee that the cells fit together properly to form a surface everywhere.

In this survey talk, I will describe the underlying mathematics that forms the basis for topological editing of maps, and I will sketch the algorithms that have been used to implement tests that are based on that mathematics. As an example of possible new research opportunities, I will present a very useful duality property in map overlay analysis, which I will prove by evaluating the Mayer- Vietoris homology sequence applied to two map layers.

Lisbeth Fajstrup
"Dicoverings"
May 4, 2001

Higher Dimensional Automata, HDA, were introduced in [4] as models for concurrent systems. They were interpreted as cubical complexes in [3], A distinguishing feature for HDA's is a partial order reflecting the direction of time. In particular, paths modelling the execution of a program, are increas- ing with respect to this local time direction. It is the goal of \ditopology" to study the effects of this extra timedirection from a topological perspective. To this aim, locally partially ordered spaces were introduced and investigated in [1] and [2].

This talk focuses on one aspect of this \ditheory", namely coverings. We define a diconnected (universal) covering, and we give a first definition of dicoverings in general for a subcategory of the category of local po-spaces. From a computer scientific point of view, the interest in these coverings come from the study of \ deloopings". The universal dicovering is the largest - infinite- delooping. However, it is not clear that the lower deloopings can be considered as dicoverings.

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Afra Zomorodian -University of Illinois
"Topological Persistence"

Persistence measures the length of the lifetime of a topological change in a filtration, based on homology theory. Using persistence, we may simplify a filtration topologically, eliminating the "noise" and retaining the features. We have fast algorithms for computing persistence and topological simplifications. I will present evidence of the speed and utility of my implementations. I will also briefly discuss applications of persistence to molecular feature detection (using alpha shapes), grid simplification (using Morse complexes), and linking number computation.

This is joint work with Herbert Edelsbrunner and John Harer (Duke), and David Letscher (Oklahoma State). It constitutes part of my doctoral work.

Lowell Abrams* and Dan Slilaty
"A topological approach to graph embeddability"

Using the theory of simplicial homology, we provide new proofs ofMacLane's and Whitney's criteria for graph planarity, and extend these to criteria for embeddability in the projective plane. We construct a framework for understanding graph embeddability in surfaces of higher genus and demi-genus.

Martin Raussen - Aalborg University
"State spaces and dipaths up to dihomotopy"

V.Pratt \cite{Pratt} and \'{E}.Goubault \cite{Goubault} introduced and investigated higherdimensional automata as geometric models for concurrent processes. In these models, a computation corresponds to an oriented path (\emph{dipath}) and \emph{di}homotopic dipaths correspond to equivalent computations \cite{FGR:99}.
As an algebraic topologist, one is tempted to search for counterparts in this framework to the components of a state space, to its homotopy groups and to the homomorphisms induced by a \emph{di}map. This talk advocates the use of categories of fractions of the fundamental category of the state space to define and to reason about these concepts, and to give first results and calculations about the category of dicomponents, compare \cite{Raussen:00, Sokolowski:00, Sokolowski:01}. For concurrency applications, this vorresponds to a drastical reduction of the state space to be considered.

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Masahiro Hachimori
"Decompositions of 2-dimensional simplicial complexes"

There are combinatorial concepts of decomposing simplicial complexes recursively, such as shellability and constructibility (weaker concept than shellability), etc. Very few facts are known about which simplicial complexes admit such decomposition. In 3- and higher dimensional cases, the existence of nonshellable (resp. nonconstructible) triangulations of balls and sphres make it very complicated to determine whether a given simplicial complex is shellable (constructible) or not. In the 2-dimensional cases, if the underlying space is a surface, shellability can be easily determined from its topology, that is, a simplicial complex is shellable iff it is a disk or a sphere. But for general simplicial complexes, the problem is still difficult even if restricted to 2-dimensional cases.
Starting from the known fact that any triangulation of contracible polyhedron with no boundary, such as the dunce hat, is not shellable, we show that some of them are constructible while some are not constructible, then give a class of polyhedron whose triangulations are nonconstructible. This suggests that the topology of the polyhedron gives some information for the decomposability. To discuss what topology allows such decomposition, we characterize the class of 2-polyhedra which admit shellable triangulations. Such characterization for constructibility is still open.

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"Computation of Cohomology Operations on Finite Simplicial Complexes"

We propose here a method for calculating (co)homology invariants of a finite simplicial complex.
Of course, there exist methods for computing homology groups, for example, the classical algorithm that consists in constructing matrices that correspond to the dierential in each degree and reducing them to the Smith normal form, from which one can read ohomology groups of the complex [Mun84] or the incremental algorithm for computing Betti numbers [DE93]. However, we haven’t found in the literature any real algorithm for computing cohomology operations.
We sketch a general method that includes the computation of (co)homology groups, some primary

cohomology operations and secondary cohomology operations, cup–i products and A1 structure of a finite simplicial complex X.
This method is based on the transcription of the classical algorithm for calculating (co)homology groups mentioned above, in terms of an special type of algebraic homotopy equivalence, called contraction, from the (co)chain complex of X onto a “minimal” (co)chain complex M(X). Using this contraction and our algorithms for computing cup–i products at cochain level developed in [GR99a] and [GR99b], it is easy to calculate the image of Steenrod squares at cohomology level and the study of cup–i products of cocycles. In a last step, we show an algorithm for calculating the Adem secondary cohomology operation 1 from the Kernel of Sq2(H2(X)) onto H5(X)/Sq2H3(X) and we give a first analysis of its complexity.

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*Speaker

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