SYLLABUS FOR THE REAL ANALYSIS QUALIFYING EXAM

Stanford University Mathematics Department

Set Theory. Countable and uncountable sets, the axiom of choice, Zorn's lemma.

Metric spaces. Completeness; separability; compactness; Baire category; uniform continuity; connectedness; continuous mappings of compact spaces.

Functions on topological spaces. Equicontinuity and Ascoli's theorem; the Stone-Weierstrass theorem; topologies on function spaces; compactness in function spaces.

Measure. Measures and outer measures; measurability and σ-algebras; Borel sets; extension of measures; Lebesgue and Lebesgue-Stieltjes measures; signed measures; absolute continuity and singularity; product measures.

Measurable functions. Properties of measurable functions; approximation by simple functions and by continuous functions; convergence in measure; Egoroff's theorem; Lusin's theorem; Jensen's inequality.

Integration. Construction and properties of the integral; convergence theorems; Radon-Nykodym theorem; Fubini's theorem; mean convergence.

Special properties of functions on the real line. Monotone functions; functions of bounded variation and Borel measures; absolute continuity; differentiation and integration; convex functions; semicontinuity; Borel sets; properties of the Cantor set.

Elementary properties of Banach and Hilbert spaces. $\mathcal{L}^p$ spaces; $C(X)$; completeness and the Riesz-Fischer theorem; orthonormal bases; linear functionals; Riesz representation theorem; linear transformations and dual spaces; interpolation of linear operators; Hahn-Banach theorem; open mapping theorem; uniform boundedness (or Banach-Steinhaus) theorem; closed graph theorem; Alaoglu theorem.

Basic harmonic analysis. Basic properties of Fouries series and the Fourier transform; Poission summation formula; convolution; mollification.

References

The above topics are covered in the following:

(1) Royden, *Real Analysis*, except chapters 8, 13, 15.
(2) Dym and McKean, *Fourier Series and Integrals*, chapters 1 and 2, or Katznelson, *Introduction to harmonic analysis*, chapters 1, 2, and 4.
(3) Gilbarg and Trudinger, *Elliptic Partial Differential Equations of Second Order*, chapter 7.2 and 7.3.