SYLLABUS: STANFORD PHD QUALIFYING EXAM IN ALGEBRA

I. GROUP THEORY. Group actions, Sylow Theorems and applications, symmetric and alternating groups, simple groups, Jordan-Holder theorem, linear groups (GL(n, F) and its subgroups), p-groups, solvable and nilpotent groups, group extensions, semi-direct products, free groups, amalgamated products and group presentations.

II. COMMUTATIVE RINGS. Principal ideal domains, unique factorization domains, polynomial and power series rings, prime and maximal ideals, Chinese Remainder Theorem, local rings and localization, Nakayama's lemma, Noetherian rings, Hilbert basis theorem, Artin rings, integral ring extensions, Dedekind domains, Nullstellensatz, algebraic sets, Spec(A).

III. MODULES. Finitely generated modules over PID's and applications, chain conditions (especially Noetherian modules), projective and injective modules, tensor products, symmetric and exterior powers, Hom, Tor and Ext.

IV. FIELD THEORY. Degree of an extension, algebraic and transcendental field extensions, splitting fields and algebraic closure, finite fields, normal and separable extensions, theorem of the primitive element, inseparable extensions, field embeddings and automorphisms, Galois theory, norms and traces, cyclic extensions, Galois theory of number fields, transcendence degree, function fields.

V. LINEAR ALGEBRA. Characteristic and minimal polynomials, Jordan and rational canonical forms, eigenvalues and eigenvectors, trace and determinant, diagonalization, multilinear algebra, bilinear forms, orthogonal groups, spectral theorem (real and complex cases).

VI. NON-COMMUTATIVE RINGS AND GROUP REPRESENTATIONS. Irreducible representations, Schur's lemma, Maschke's theorem, characters, Schur orthogonality, character tables, semisimple group rings, induced representations, Frobenius reciprocity, tensor products, symmetric and exterior powers, complex, real, and rational representations, division rings, matrix rings, semisimple rings and modules.