ABSTRACT:
Diagonal Harmonics are the polynomials $P(x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_n)$ that are killed by the differential operators $\sum_{i=1}^{n} \frac{\partial^r}{\partial x_i^r} \frac{\partial^s}{\partial y_i^s}$ (for $0 < r + s < n$). This is a finite dimensional space used by Garsia-Haiman to give a representation theoretical setting to the Macdonald polynomials. Since its discovery, combined efforts by a number of researchers in algebraic combinatorics revealed intimate connections of its representation theoretical properties to a variety of combinatorial constructs. Parking functions, originally introduced by computer scientists, emerged from the very beginning as the most effective combinatorial structure to provide insight into the structure of the Diagonal Harmonics. Unproven for nearly a decade, the “Shuffle Conjecture” asserts that one can calculate the Frobenius characteristic of the Diagonal Harmonics by $q,t$-counting parking functions by area and dinv (with an appropriate quasi-symmetric function). In a 2010 paper, Haglund, Morse, and Zabrocki sharpened the Shuffle Conjecture, thereby providing a new approach to the problem. In this talk we show how a conjectured property of a certain family of polynomials in an single variable can be used to establish an increasing number of special cases of the Haglund, Morse, and Zabrocki Conjecture and in particular obtain a computer proof of the Shuffle Conjecture up to $n = 14$. 