Abstract: Elliptic boundary value problems are well-understood in the case when the boundary, the data, and the coefficients exhibit smoothness. However, perfectly uniform smooth systems do not exist in nature, and every real object inadvertently possesses irregularities (a sharp edge of the boundary, an abrupt change of the medium, a defect of the construction).

The analysis of general non-smooth elliptic PDEs gives rise to decisively new challenges: possible failure of maximal principle and positivity, breakdown of boundary regularity, lack of the classical $L^2$ estimates, to mention just a few. Further progress builds on an involved blend of harmonic analysis, potential theory and geometric measure theory techniques. In this talk we are going to discuss some highlights of the history, conjectures, paradoxes, and recent discoveries such as the higher-order Wiener criterion and maximum principle for higher order PDEs, solvability of rough elliptic boundary problems, as well as an intriguing phenomenon of localization of eigenfunctions—within and beyond the limits of the famous Anderson localization.

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