Variation with $p$ of the number of solutions mod $p$

of polynomial equations

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Abstract

Let $f = (f_1, f_2, \ldots)$ be a family of polynomials in several variables, with coefficients in $\mathbb{Z}$. If $p$ is a prime number, let $N(f; p)$ be the number of solutions mod $p$ of the system of equations $f(x) \equiv 0 \mod p$. We shall discuss the way $N(f; p)$ varies with $p$: closed formulae, computability, size, congruence properties, relations with topology, etc.